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“Igor Sikorsky Kyiv Polytechnic Institute”

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THEORY OF MECHANISMS AND MACHINES

PART 2:

SYNTHESIS OF MECHANISMS, FRICTION, VIBRATION PROTECTION

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as a textbook for students studying for specialty
"Applied Mechanics"

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Electronic Networking Training Edition

Zakhovaiko O. P.

Theory of mechanisms and machines. In 2 parts. [Electronic resource]: Textbook / O. P. Zakhovaiko. – Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2021 . – Part 2. : Synthesis of mechanisms, friction, vibration protection. – Electronic text data (1 file: 9,13 MB). – 184 p.

Part 2 of the textbook deals with the synthesis of mechanisms with lower and higher kinematic pairs. It examines the basic principles of the theory of gearing and cam mechanisms, methods of their analysis and synthesis. This part of text book considers the effect of friction in kinematic pairs, peculiarities of its consideration in dynamics problems, issues of wear of elements of kinematic pairs and evaluation of their wear resistance. General analysis of the causes of vibrations in mechanisms and the basic methods of vibration protection is given. The theoretical material is accompanied by examples of its practical application and questions for self-examination. For students of speciality 131 "Applied Mechanics", subject areas "Dynamics and strength of machines" and "Information Systems and Technologies in aircraft construction".

У частині 2 підручника викладені питання синтезу механізмів з нижчими і вищими кінематичними парами. Приведені основні положення теорії зубчастого зачеплення та кулачкових механізмів, методи їх аналізу і синтезу. Розглянуті питання тертя в кінематичних парах, особливості його врахування в задачах динаміки, питання зношування елементів кінематичних пар та оцінки їх зносостійкості. Приводиться загальний аналіз причин вібрацій, що виникають в механізмах, та основні способи віброзахисту. Теоретичний матеріал супроводжується прикладами його практичного застосування, запитаннями для самоперевірки. Для студентів спеціальності 131 «Прикладна механіка», спеціалізації «Динаміка і міцність машин» та "Інформаційні системи та технології в авіабудуванні".

CONTENTS

FOREWORD

The textbook “Theory of mechanisms and machines: synthesis of mechanisms, friction, vibration protection” is the second part of the textbook on academic discipline “Theory of mechanisms and machines” and is intended for students of speciality 131 "Applied Mechanics", subject areas "Dynamics and strength of machines" and "Information Systems and Technologies in aircraft construction".

Part 2 of the textbook is devoted to the consideration of problems of synthesis of mechanisms with lower and higher kinematic pairs. It describes the basic principles of the theory of gearing and cam mechanisms, friction in kinematic pairs and evaluation of their wear resistance, problems of vibroprotection of mechanisms and machines.

Part 2 of the textbook contains an introduction and five chapters. Their numbering is a continuation of the numbering of the s chapters included in part 1 of the textbook – from chapter 8 to chapter 12.

Chapter 8 covers the basics of mechanism synthesis. Theoretical material is accompanied by examples of implementation of methods of synthesis of hinged-lever mechanisms, as well as formation of mating surfaces of links in mechanisms with higher pairs.

Chapter 9 discusses the synthesis of gearings and their current classification. The focus is on involute gearing, which is a staple among modern mechanical gears. Here, their features, manufacturing methods and quality control are considered. General information on spatial gears, in particular bevel and hyperboloid gears, is also provided.

Chapter 10 presents the theory of cam mechanisms. Here, analytical and graphoanalytical methods for their synthesis and analysis are considered.

Chapter 11 deals with the basics of dry friction theory and the theory of lubrication in kinematic pairs. The influence of friction on the operation of machines and mechanisms is analysed. In addition, the issues of wear in kinematic pairs and the wear resistance of their elements are considered.

Chapter 12 discusses the issues of vibration in mechanisms, analyses the main sources of their occurrence. Methods of vibration protection of mechanisms and machines are also considered, examples of design of dampers, their types and applications are given.

Each chapter of the textbook is accompanied by questions for students to self-test knowledge.

The textbook contains a number of appendixes with reference information for use in solving practical problems and in self-testing of knowledge, as well as an English-Ukrainian terminological dictionary.

Chapter 8. SYNTHESIS OF PLANAR MECHANISMS

When designing mechanisms schemes (synthesis) by a chosen structure chart and set kinematic parameters we should define dimensions of links of mechanisms, which can provide necessary motion.

8.1. SYNTHESIS OF KINEMATICS SCHEMES OF MECHANISMS WITH LOWER KINEMATIC PAIRS

Here we are talking about hinged-lever mechanisms. Their kinematic synthesis consists of the set of concrete tasks:

- synthesis by several positions of links;
- synthesis by some set kinematic parameters (for example average speed);
- synthesis by a set path of a link's point.

8.1.1. Crank existence condition for a pin-jointed four-bar linkage (Grashof's criterion)

The linkages that were discussed hitherto contained cranks. That is, they each contained a link that was capable of rotating through 360º relative to fixed frame. Therefore, each of them could be driven by a continuously rotating shaft.

Franz Grashof (1826–1893)
German engineer and scientist, professor of Applied Mechanics at the Karlsruhe Institute of Technology, a specialist in the field of kinematics of mechanisms, strength of materials. He made a significant contribution to the development of hydraulics and thermodynamics, developed analytical methods for kinematic study of mechanisms, formulated a theorem on velocity projections and on a four-bar hinged-lever mechanism. He was one of the founders of the Association of German Engineers, which still exists today.

When carrying out synthesis of such mechanism one of essential requirement is possibility of rotating of its links, in other words, there should be one or more cranks in its scheme. This possibility depends on aspect ratio of links.

Let us study a pin-jointed four-bar linkage with lengths of links *a*, *b*, *c* and *d*. (Fig. 8.1)

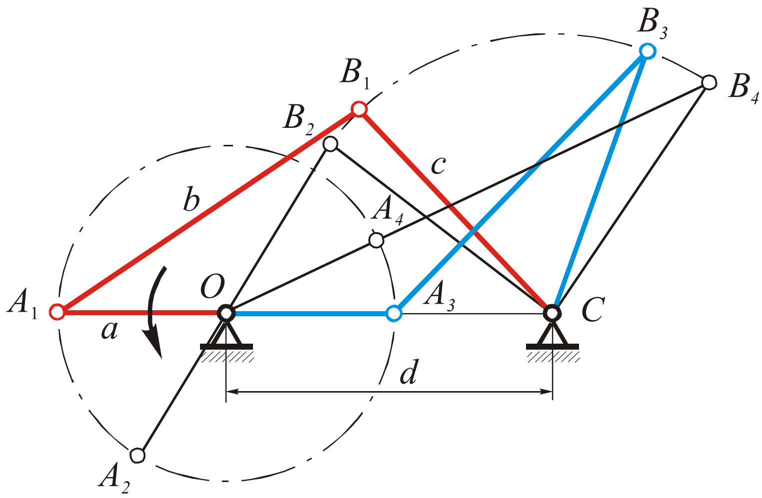


Fig. 8.1. Pin-jointed four-bar linkage

For the *OA* link to be a crank it should pass through two extreme positions: OA_1 and OA_3.

Assume that the dimension *a* of the *OA* link is minimal, and the dimension *d* (fixed *OC* link) is maximal. Then we study a triangle A_1B_1C. As in the triangle the length of one side is less than sum total of lengths of the other two sides, we write down such inequality:

$$d + a < b + c\,. \tag{8.1}$$

For a triangle A_3B_3C the following condition is true:

$$d - a < b + c\,. \tag{8.2}$$

As by the condition $d > a$, so inequality (8.1) provides fulfilment of inequality (8.2) independently from ratio of side dimensions *b* and *c*. If the link *AB* ($b > c > d$) or *BC* ($c > b > d$) is the longest, so inequality (8.1) is fulfilled.

Inequality (8.1) helps to formulate Grashof's criterion.

The minimal link of a pin-jointed four-bar linkage can be a crank, if the sum of lengths of the maximal and minimal links of the mechanism is less than the sum of lengths of other links.

Positions of the crank OA_2 and OA_4 correspond to extreme positions of a rocker CB.

8.1.2. Examples of synthesis of hinged-lever mechanisms

Synthesis of a rocker-and-crank mechanism by two positions of links. By set distance between pivots O and C, which belong to the fixed frame (Fig. 8.2), length of a rocker 3 l_3 and its angular data γ_1 and γ_2 in extreme positions of the mechanism we should find necessary lengths of the crank 1 l_1 and the connecting rod 2 l_2.

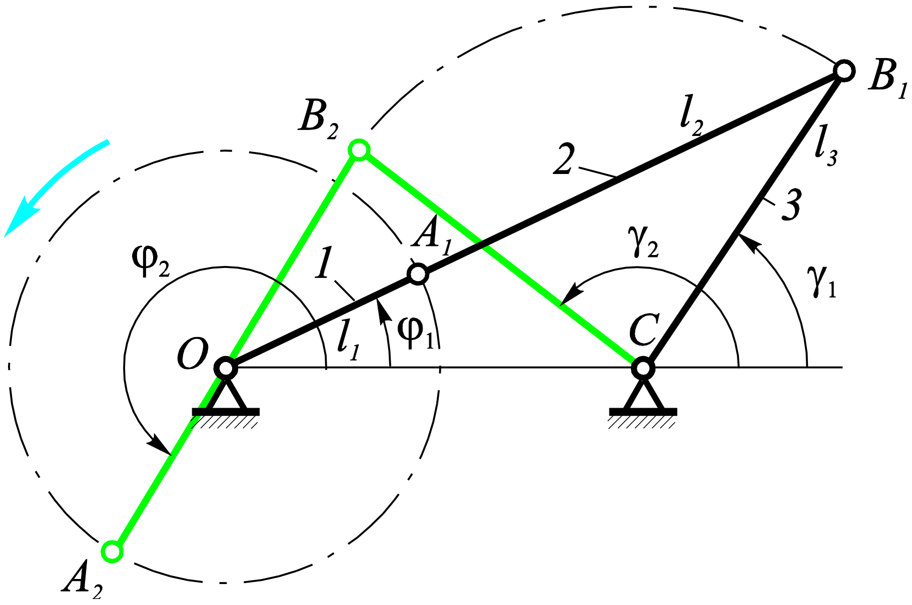


Fig. 8.2. Synthesis of a rocker-and-crank mechanism

For this we connect points B_1 and B_2 with a point O by straight lines. We have

$$l_{OB_1} = l_1 + l_2; \; l_{OB_2} = l_2 - l_1.$$

Hence

$$l_1 = \left(l_{OB_1} - l_{OB_2}\right)/2; \qquad l_2 = \left(l_{OB_1} + l_{OB_2}\right)/2.$$

Synthesis of a crank-and-slider mechanism by an average speed of an output link. When designing machines the average speed of a slider V_c (m/s) is often set. Let us study the in-line slider-and-crank mechanism (Fig. 8.3).

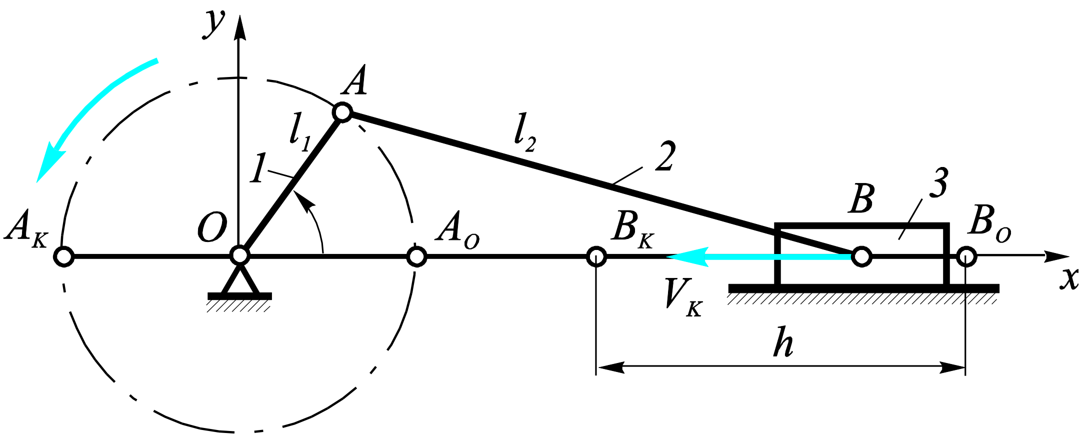


Fig. 8.3. Synthesis of a crank-and-slider mechanism

Here, the double stroke of the slider 3 corresponds to one full revolution of the crank 1. We can write down

$$2h = 4l_1.$$

If turning speed of a crank (number of revolutions per minute) is denoted by n, then

$$v_c = 2hn = 4l_1 n.$$

Hence, length of a crank

$$l_1 = v_c / 4n.$$

Length of a connecting rod 2 is chosen, by setting the ratio $\lambda_2 = l_2 / l_1$. The less the ratio is, the less dimensions of a mechanism we have, but bigger pressures appear in unfavourable positions of a mechanism in kinematic pairs. For example, for mechanisms of internal-combustion engine we choose this ratio in the range $\lambda_2 = 3...5$.

8.2. SYNTHESIS OF MECHANISMS WITH HIGHER KINEMATIC PAIRS

8.2.1. The fundamental law of gear tooth action (The main theorem for gearing)

The advantages of these mechanisms are: possibility to realize necessary laws of motion by a minimum of links and comparatively higher playback accuracy of these laws in comparison with hinged-lever mechanisms.

Surfaces of higher pairing elements, which provide a prescribed law of motion of links, are called mating surfaces.

Mechanisms with higher pairs can have one pair of mating surfaces (profiles), for example, cam mechanisms, or several pairs, as in wheelworks.

There is interrelation between geometry of mating profiles and law of relative motion of higher pairing elements. It is established by the *fundamental law of gear tooth action* or *the main theorem for gearing*.

In problems of synthesis of mating profiles the law of relative motion is set. Really, in relationship

$$\bar{\omega}_2 = \bar{\omega}_1 + \bar{\omega}_{21}$$

velocities $\bar{\omega}_1$ and $\bar{\omega}_2$ are known. Hence

$$\bar{\omega}_{12} = \bar{\omega}_1 - \bar{\omega}_2 .$$

In Fig. 8.4 there are examples, in which we see that velocity vectors $\bar{\omega}_1$ and $\bar{\omega}_2$ can be parallel, intersect or skew.

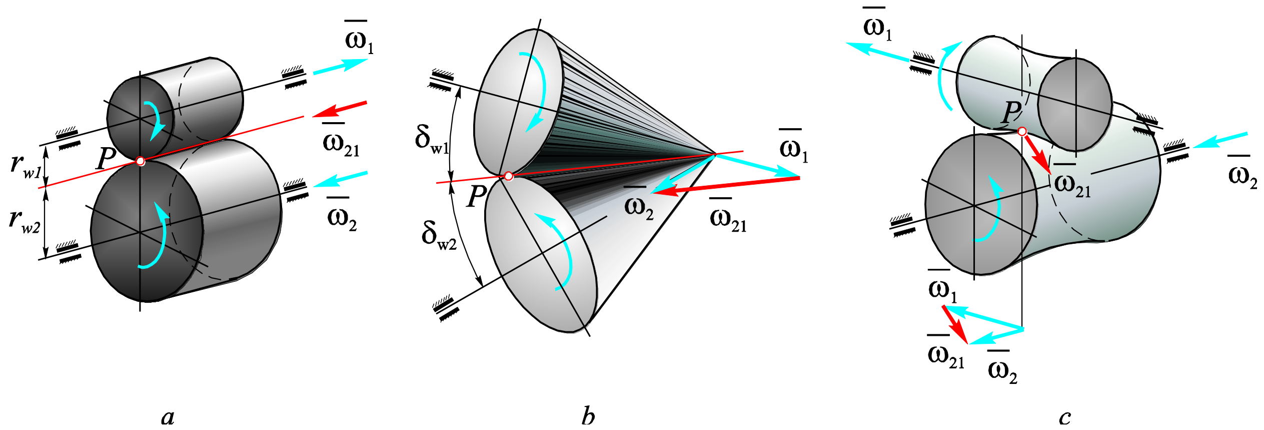


Fig. 8.4. Types of axoids: *a* – cylinders; *b* – cones; *c* – hyperboloids

The axis, which traverses point P, is an instant axis of one of the links in coordinate system connected with another link.

Axoid is locus of instant axes in a primary reference system.

When axes of rotations are fixed and parallel, axoids are cylinders (Fig. 8.4, *a*), which are tangent to moving line and roll over each other without sliding.

When axes of rotations are fixed and intersect in a certain point (Fig. 8.4, *b*), axoids are cones with vertex angles $2\delta w_1$ and $2\delta w_2$. These angles define position of an instant axis in a primary reference system.

When axes of rotations skew (Fig. 8.4, *c*), relative motion of links is helical or screw motion, so to say it consists of turning motion about some axis and sliding motion along this axis. In such case we should say about *instant screw axis*. If angular velocities of links $\overline{\omega}_1$ and $\overline{\omega}_2$ are steady speed, so axoids are hyperboloids of rotation with rectilinear generator, which roll over each other, being tangent in instant screw axis, and sliding along this axis.

In general case fundamental law of gear tooth action can be formulated as:

Mating profiles in arbitrary contact point have common normal to their surfaces, which is perpendicular to the direction of velocity of contact point in an assigned relative motion of these surfaces.

It is very easy to prove this theorem by going from the contrary assertion: non-perpendicularity of common normal in a contact point of mating surfaces to the direction of relative velocity vector requires one more velocity component –

in the direction to this common normal. And in this case higher pairing elements should either interpenetrate, or lose contact, which contradicts criterion of higher kinematic pair creation.

8.2.2. The fundamental law of gear tooth action for planar mechanisms (Willis′ theorem)

For planar mechanisms with higher pair we use the term *instantaneous centre of rotation*.

Centrode or centroid line is locus of instantaneous centres of rotation of links at relative motions in the primary reference system.

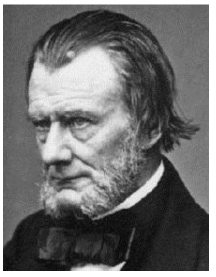
Centrode is formed as an intersection of axoid with crosscut secant plane. As well as axoids, centrodes roll over each other without sliding.

Tangent point of centrodes P is called *pitch point* (See Fig. 8.4, *a*).

The fundamental law of gear tooth action for planar mechanisms (Willis′ theorem) is formulated as:

A common normal in contact point of mating profiles in any moment of tooth action passes pitch point P and divides axle base O_1O_2 in inverse proportion to angular velocity ratio of links.

Robert Willis (1800–1875)
Professor of the University of Cambridge. He worked in the field of mechanics and acoustics of human speech. In his most notable work, "Fundamentals of Mechanisms," a book that earned him general recognition in the technical sciences, he outlined his ideas about mechanisms and proposed a new system for classifying them. He also developed the theory of the form of gear teeth.



To prove this theorem we use Fig. 8.5.

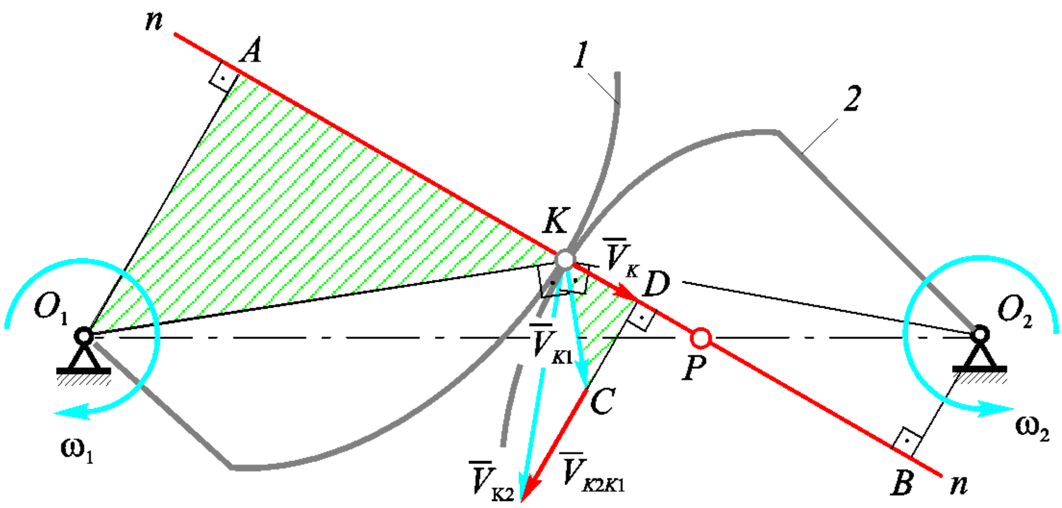


Fig. 8.5. Planar mechanism with a higher pair

Here $\overline{V}_{K1}$ and $\overline{V}_{K2}$ are absolute velocities of the point *K* for the first and second links respectively. For them the following equation is true:

$$\overline{V}_{K2} = \overline{V}_{K1} + \overline{V}_{K2K1}.$$

According to fundamental law of gear tooth action, velocity of the point *K* in the direction of common normal *n-n* for both links is equal: $\overline{V}_{K1}^{n} = \overline{V}_{K2}^{n} = \overline{V}_K$.

From the similarity of triangles O_1AK and *KDC* we can write down the proportion:

$$\frac{O_1A}{O_1K} = \frac{KD}{KC}. \tag{8.3}$$

Considering that $KD = \left|\overline{V}_K\right| = \left|\overline{V}_{K2}^{n}\right| = \omega_2\left(\mu_l \cdot O_2B\right)$ and $KC = \left|\overline{V}_{K1}\right| = \omega_1\left(\mu_l \cdot O_1K\right)$, we rewrite proportion (8.3) as:

$$\frac{O_1A}{O_1K} = \frac{\omega_2\left(\mu_l \cdot O_2B\right)}{\omega_1\left(\mu_l \cdot O_1K\right)}$$

or

$$\frac{O_1A}{O_2B} = \frac{\omega_2}{\omega_1}.$$

Now we study triangles O_1AP and O_2BP. They are similar, thus

$$\frac{O_1A}{O_2B} = \frac{O_1P}{O_2P}.$$

So to say

$$\frac{\omega_2}{\omega_1} = \frac{O_1P}{O_2P} = u_{21},$$

that is what we had to prove.

According to fundamental law of gear tooth action for planar mechanisms a position of pitch point P is uniquely defined through axle base O_1O_2 (the line connecting the centers of rotation of links) and velocity ratio u_{21}, which should be set in synthesis problems.

8.2.3. Graphical methods of mating profiles synthesis

8.2.3.1. Method of consecutive positions of profiles

The method is based on the kinematic inversion principle, according to which one of centrodes ($Ц_2$ in Fig. 8.6) stops, and the ground together with another centrode ($Ц_1$) is turning with the velocity $-\omega_2$. The centrode $Ц_1$ carries out relative turning motion around the pitch point P with the velocity ω_{12}.

The method consists in plotting a sequence of serial positions of the profile $П_1$ and building profile $П_2$ as inside envelope of these positions (Fig. 8.6).

On centroids the points by which they touch at rolling are shown: respectively 1^* and 1^{**}; 2^* and 2^{**} etc. As centrodes roll without sliding, positions of relative points are defined using the equality conditions of matched arcs of centrodes: $\cup P1^* = \cup P1^{**}$; $\cup P2^* = \cup P2^{**}$; ...; $\cup P10^* = \cup P10^{**}$.

In inverse motion the line, which connects centres of rotation of centrodes O_1O_2, will take serial positions O_2-1; O_2-2; ...; O_2-10. Points 1^{**}, 2^{**}, ..., 10^{**} of a fixed centrode $Ц_2$, which are instantaneous centre of the centrode $Ц_1$ rotation, are considered as pitch points at matched positions.

When the pitch point is in the point 2^{**}, so to say centrode $Ц_1$ takes position $Ц_1^{(2)}$ (Fig. 8.6, *b*), the ray O_12^* coincides with centre line O_2-2. At the same time centrode $Ц_1$ turns by angle $\varphi^{(2)}$ (Fig. 8.6). Then we attach profile $П_1$ to the line O_1P (initial position of the construction). In the position of centre line O_2-2 the profile takes position *2*. The Fig. 8.6 shows the position of the profiles in the inverse motion in circles.

Then we construct a required mating profile $П_2$ as an envelope of obtained serial positions of the assigned profile $П_1$. In the Fig. 8.6 this profile is emphasized by hatching.

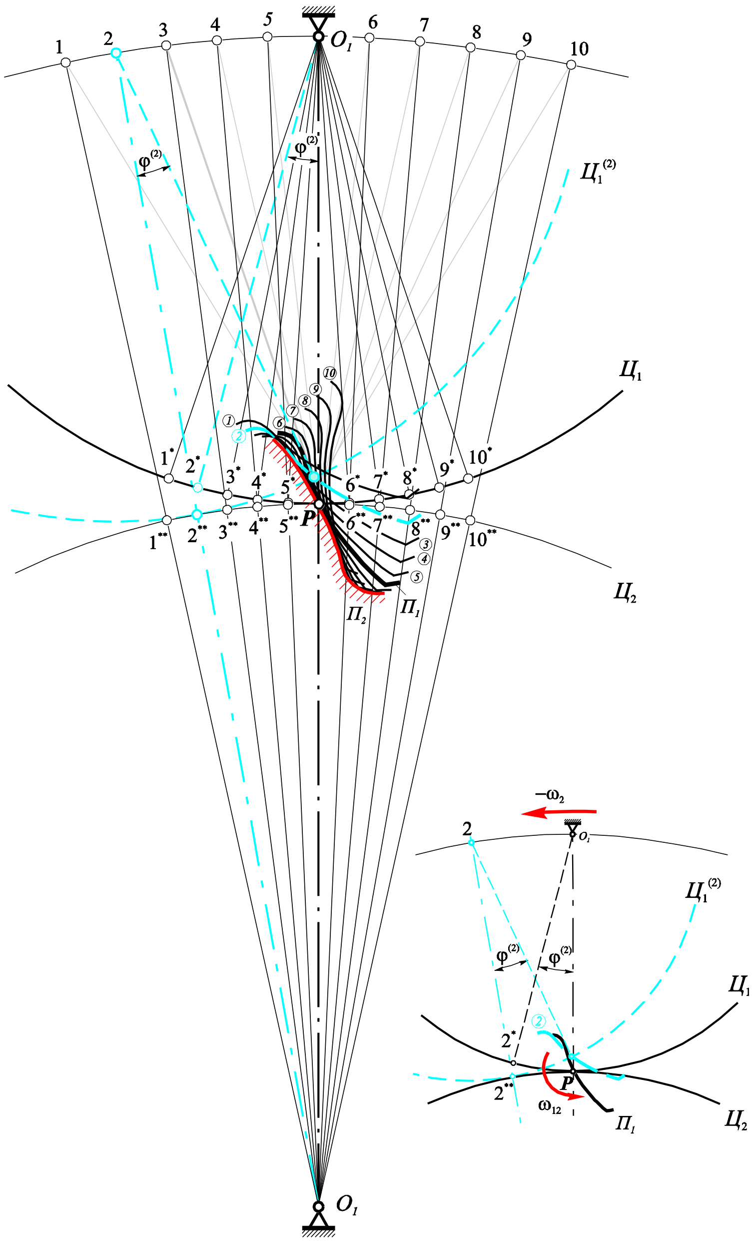
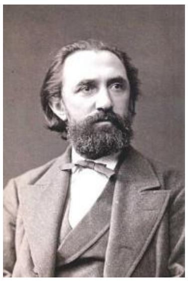


Fig. 8.6. Mating profiles synthesis by the method of consecutive positions of profiles

8.2.3.2. Method of consecutive positions of normal (Reuleaux method)

Mating profile can be constructed by positions of normal. This method is based on the fundamental law of gear tooth action. It is rather effective in cases when we can easily define positions of normal to the assigned profile.



Franz Reuleaux (1829–1905)
German scientist, a specialist in the field of mechanical engineering. He worked at the Swiss Federal Institute in Zurich, later at the Berlin Royal Technical Academy, and was its president. F. Reuleaux made an outstanding contribution to the development of the theory of mechanisms and machines, formulated the concept of a kinematic pair and built the doctrine of the mechanisms based on it. He developed methods for the synthesis of mechanisms.

Let's consider as set the axle base O_1O_2, the law of relative motion of links $u_{21} = \omega_2 : \omega_1 = O_1P : O_2P$ and profile $П_2$ (Fig. 8.7). Then we construct profile $П_1$ mating with it.

We choose on the profile $П_2$ a set of points $1_2, 2_2, 3_2, ... 6_2$. Then we draw normals to the profile through these points to the crossing with the centrode $Ц_2$ in the points 1^{**}, 2^{**}, 3^{**}, ..., 6^{**} respectively. Then we put points 1^*, 2^*, 3^*, ..., 6 on the centrode $Ц_1^*$, with which matched points of centrode $Ц_2$ make a contact when passing the pitch point P. For this we use conditions: $\cup P1^* = \cup P1^{**}$; $\cup P2^* = \cup P2^{**}$; ...; $\cup P6^* = \cup P6^{**}$, as centrodes roll without sliding.

According to the fundamental law of gear tooth action the contact point of mating profiles is on the normal, which passes pitch point. Proceeding from this, we can find positions of points of the profile $П_2$ on the fixed plane, when they are in contact with the profile $П_1$. As an example we take the point 1_2. It is a vertex of triangle $O_2 1_2 1^{**}$ (Fig. 8.7, *b*). Then we turn this triangle together with centrode $Ц_2$ relatively to the point O_2 till point 1^{**} coincides with the pitch point P (Fig. 8.7, *c*). As the side of triangle $1_2 1^{**}$ coincides with normal to the profile $П_2$ in the point 1_2, so the point *1* should be studied as the contact point of profiles.

In the same way we can find positions of other contact points on the fixed plane (See Fig. 8.7, *a*).

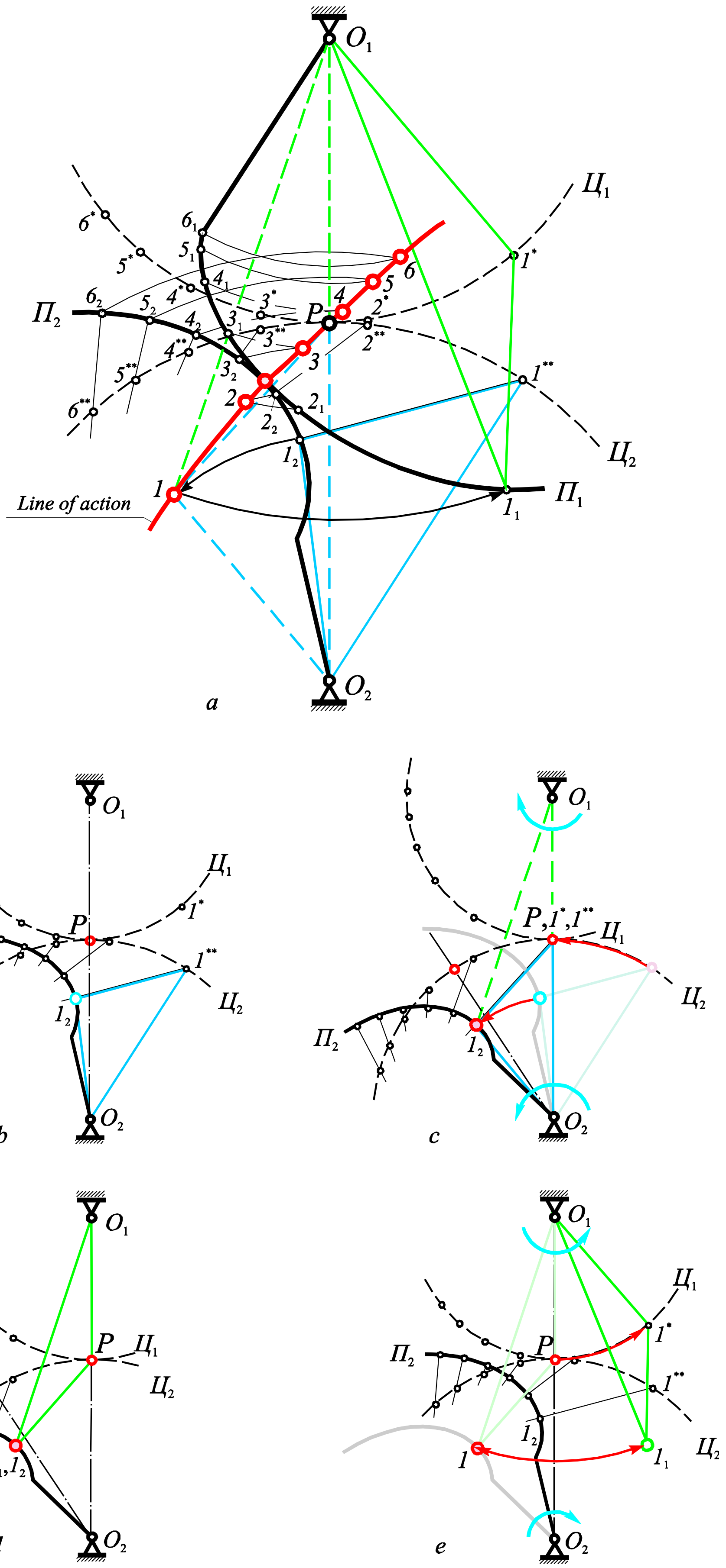


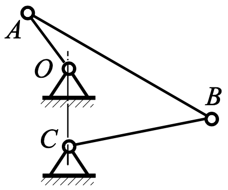
Fig. 8.7. Mating profiles synthesis by the Reuleaux method: Reuleaux method fulfilment (*a*); stages of synthesis (*b*, *c*, *d*, *e*)

Locus of contact points of mating profiles 1, 2, 3, ..., 6 is called line of action.

Normal to the profile $П_2$, which passes the pitch point, is common normal for both profiles. In the studied position, in the pitch point P the points 1^* and 1^{**} of centrodes $Ц_1$ and $Ц_2$ have met (See Fig. 8.7, *c*, *d*). By turning the centrodes to the initial position we turn simultaneously the triangle $O_1 11^*$ (Fig. 8.7, *d*, *e*). The obtained point 1_1 of the profile $П_1$ is mating with the point 1_2 of the profile $П_2$.

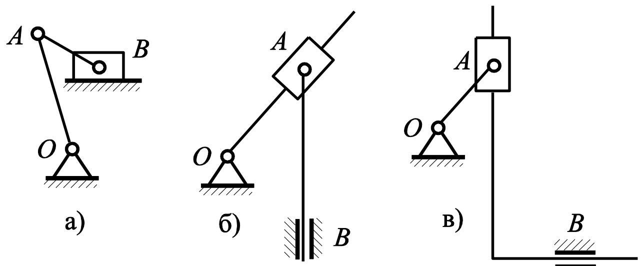
In the same way we can find other points of the profile $П_1$. Connecting the points 1_1, 2_1, 3_1, ... , 6_1 by a smooth curve we get the required profile $П_1$, mating with the set profile $П_2$ (Fig. 8.7, *a*).

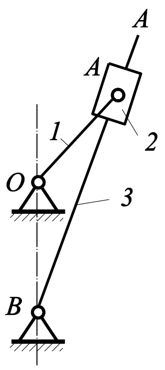
QUESTIONS FOR SELF-TESTING

1. What is the problem of mechanism synthesis?
2. What link of the mechanism is called a crank?
3. Formulate the Grashof's criterion on the existence of a crank.
4. 

   Will the *OA* link of the pin-jointed four-bar linkage carry out a full revolution if the links' lengths are: $l_{OA} = 100\ mm$, $l_{AB} = 500\ mm$, $l_{BC} = 450\ mm$, $l_{OC} = 250\ mm$

5. Is there a crank in the shown mechanisms?

6. What methods of synthesis of four-link lever mechanisms are known to you?
7. 

   What should be the length of the crank *1* of the quick-return link mechanism so that the angle of rotation of the rocker *3* between its extreme positions is $60°$. Hinge centre distance $l_{OB} = 1000\ мм$

8. What surfaces are called mating ones?
9. What surfaces are called axoids?
10. Formulate a main theorem for gearing in the general case of interaction of mating profiles.
11. What curve is called a centroid?
12. Formulate a main theorem for gearing for planar mechanisms (Willis' theorem).
13. What point of meshing is called a pitch point?

14. What principle underlies the method of consecutive positions in the synthesis of mating profiles?
15. What is the Reuleaux method of constructing mating profiles?
16. The locus of what points is the line of action?

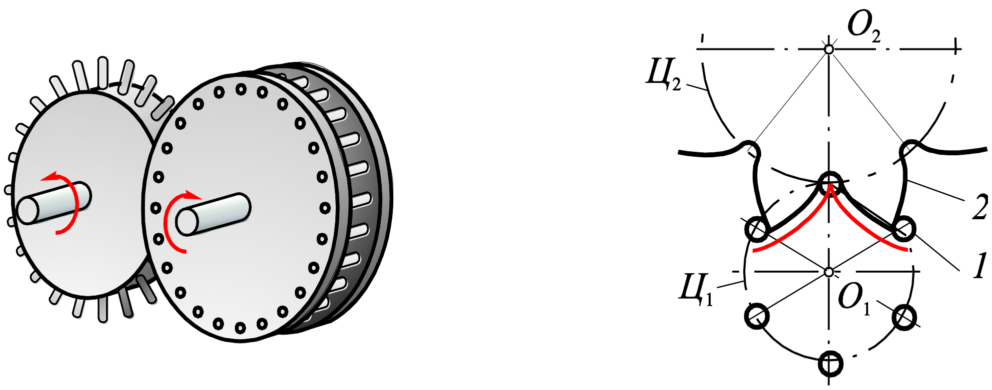
Chapter 9. GEARINGS

Gear toothing or gearing is a kind of engagement, in which continuous motion of output link is provided by serial pair-wise interaction of several mating surfaces.

The history of gearing comes from ancient Egypt, when first primitive gears were used (Fig. 9.1, *a*). They only remotely resemble today's. We may consider that direct „ancestors" of present-day gears were pin gears, which belong to cycloidal gears.

Fig. 9.1, *b* shows pin gear scheme. Cylindrical teeth (trundles) of pin wheel 1 interact with teeth of gear 2. Tooth profile of this gear made as cycloid situated equidistantly to the cycloid, built by some point of the circle of the pin wheel 1 which is a locus of centers of trundles, when rolling by the centrode of gear 2. Displacement of the cycloid for creation of a tooth profile, as is shown in the figure, is defined by the diameter of trundle.

The main disadvantage of pin gears is high wear of trundles. These gearings are not widely used.



a *b*

Fig. 9.1. Pin gear scheme: *a* – ancient prototype; *b* – present-day pin gear

The Industrial Revolution, which began in the 18th century, promoted to the search for effective means of transmission of motion. It is gears that have found the most widespread use here, which have a number of advantages: compactness under high carrying capacity, high efficiency and fidelity of reproduction of a given motion law, constancy of kinematic characteristics etc.

A real revolution in machine building happened in the middle of 19th century, when a gear hobber was invented. This invention, as an embodiment of advanced research ideas, allowed starting the mass production of high-quality machines, significantly accelerating the processes of industrialization of society.

9.1. CLASSIFICATION OF GEARINGS

At present there are various types of gearing. They differ in construction, higher pairing elements configuration, nature of executable motions. A big number of different gearings needs its classification by some attributes. Here are the main ones.

I. By type of gearing:
- involute gearing;
- noninvolute gearing (cycloidal, circular etc.).

II. By tooth forms:
- spur gearing (Fig. 9.2, *a*);
- helical gearing (Fig. 9.2, *b*);
- chevron or herringbone spur gearing (Fig. 9.2, *c*);
- with curved teeth of gears (Fig. 9.2, *d*).

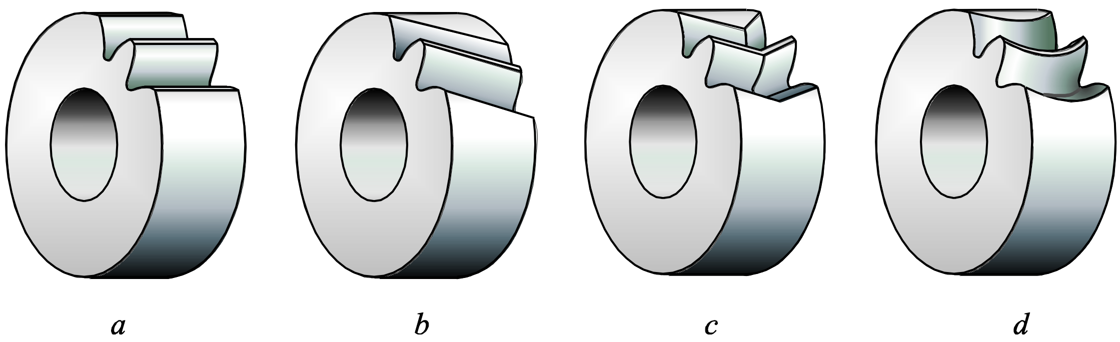
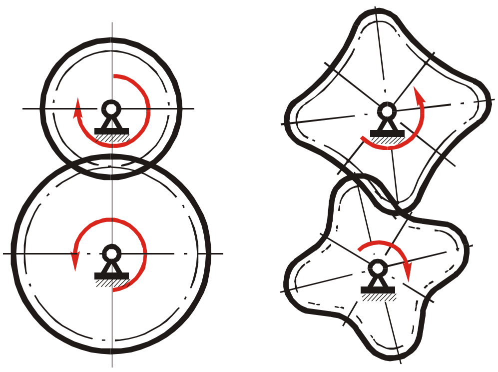


Fig. 9.2. Forms of gear teeth: *a* – spur teeth; *b* – helical teeth; *c* – chevron or herringbone teeth; *d* –curved teeth.

III. By gear forms:
- with cylindrical gears (Fig. 9.3, *a*);
- with noncylindrical gears (Fig. 9.3, *b*).



a *b*

Fig. 9.3. Forms of gears: *a* – cylindrical gears;
b –noncylindrical gears

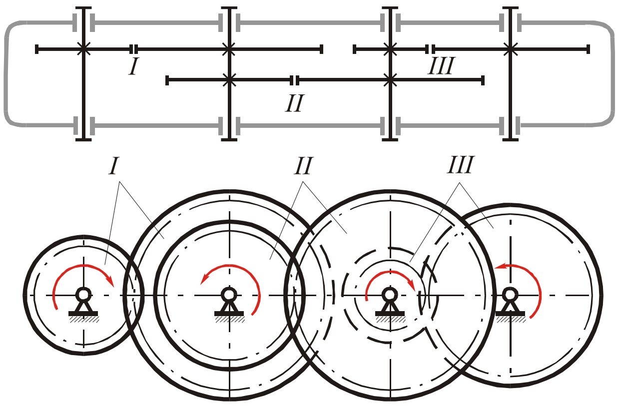


Fig. 9.4. Compound gear train

IV. By drive stages:

– single gearings or two-gear train (Fig. 9.3);

– multiple gearing or compound gear train (Fig. 9.4).

V. By arrangement of gear axes:

– with parallel axes (cylindrical gearing);

– with intersect axes (bevel gearing);

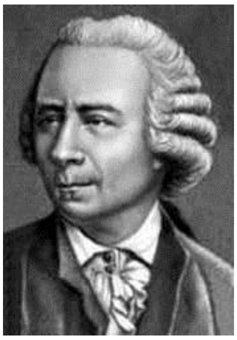
– with skew axes (worm-, screw, hypoid gearing).

9.2. INVOLUTE CYLINDRICAL GEARING

L. Euler offered to use involute as mating teeth profiles.

In toothing transmission ratio u_{12} can be both fixed and variable. In practice we often use gearing with fixed transmission ratio. Involute gearing exactly provides constancy of this ratio.

Leonhard Euler (1707–1783)
Prominent Swiss scientist, member of many European academies. He worked in Basel, Berlin, St. Petersburg, published over 850 scientific papers on mathematical analysis, differential geometry, mathematical physics, optics. He deeply studied chemistry, botany, medicine, music theory, knew many languages, including ancient ones. He is rightly considered one of the most prominent encyclopaedic scientists of all time, who laid the foundations of modern science.



9.2.1. General features

Involute is a curve that circumscribed by a point of a straight line, which rolls by circle without sliding (Fig. 9.5).

The straight line, a point of which circumscribes an involute, is tangent to circle, and at the same time it is a normal to involute profile. Points of tangency of the straight to the circle are centers of involute's curvature in the points. Thus, circle is the locus of centers of involute's curvature, i.e. *an evolute* of the involute.

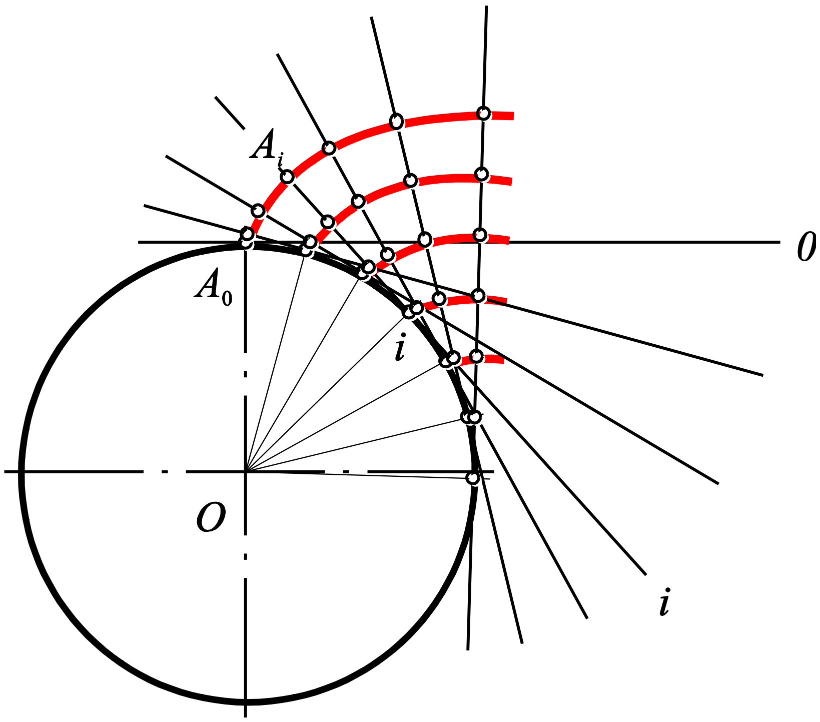


Fig. 9.5. Involute formation

Let us show that involute profiles are mated.

For this reason we superpose centers of involutes with centers of gears rotation (Fig. 9.6). The distance between them O_1O_2 is called *centre distance*. Common normal in a contact point of involute profiles is at the same time common tangent to evolutes, which is called *the base tangent*.

Let's show that point P is a pitch point. That is,

$$\frac{O_2P}{O_1P} = \frac{\omega_1}{\omega_2}.$$

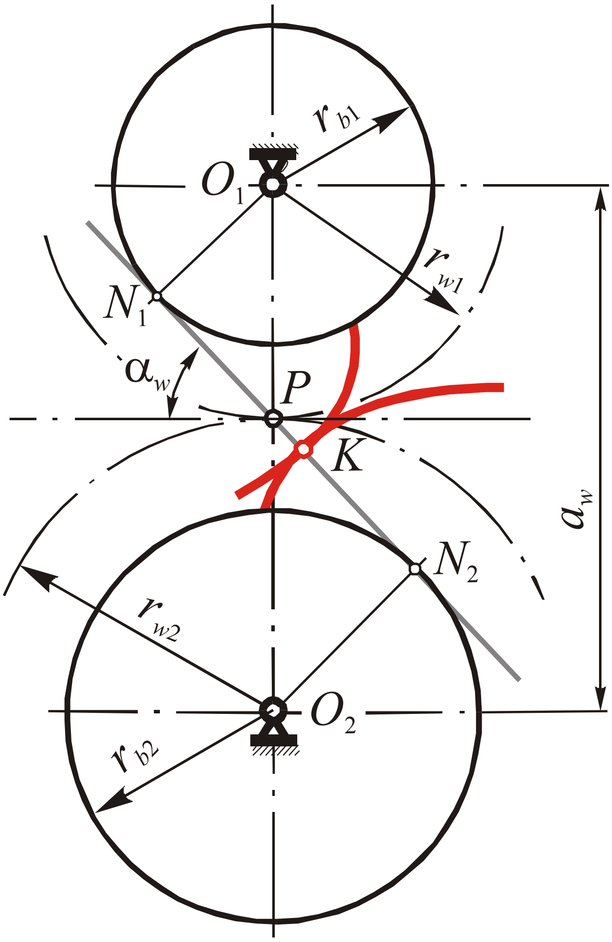


Fig. 9.6. Involute profiles

Let us draw centrodes with radiuses r_{w1} and r_{w2} through pitch point. In the pitch point, velocities of the points of the centrodes are equal, as there is no sliding: $V_1 = V_2$ or $\omega_1 r_{w1} = \omega_2 r_{w2}$.

Hence

$$\frac{\omega_1}{\omega_2} = \frac{r_{w_2}}{r_{w_1}} = \frac{O_2P}{O_1P}.$$

In such way involute profiles correspond the fundamental law of gear tooth action for planar mechanisms, i.e. they are mated, and the point P, accordingly, is the pitch point.

Involute toothing has one feature, which is very important for their application – fixed transmission ratio. Let us look at Fig. 9.6. During engagement contact point of teeth profiles K is on the straight N_1N_2, which is common tangent to evolutes of gears. No matter what is a turning angle of gears, the line N_1N_2 does not change its position, as centers of evolutes are fixed. Thus the pitch point P does not change its position on the center line O_1O_2. It means:

$$\frac{O_2P}{O_1P} = \frac{\omega_1}{\omega_2} = u_{12} = Const.$$

Let us introduce some definitions:

– the line N_1N_2 (base tangent) is called *a theoretical line of action* or *a pressure line*;

– angle between the line of action N_1N_2 and normal to the center line O_1O_2 ($\angle\alpha_w$ in Fig. 9.6) is called *pressure angle* or *angle of action*;

– circles of radiuses r_{b1} and r_{b2} – evolutes of involutes – are called *base circles*;

– circles of radiuses r_{w1} and r_{w2}, which are centrodes and are tangent in the pitch point P, are called *pitch circles*.

Pitch circle is the imaginary circle passing through the teeth of a gearwheel, concentric with the gearwheel, and having a radius that would enable it to be in contact with a similar circle around a mating gearwheel.[1]

From the similarity of triangles O_1PN_1 and O_2PN_2 we have:

$$\frac{O_1P}{O_2P} = \frac{O_1N_1}{O_2N_2} \text{ or } \quad \frac{r_{w1}}{r_{w2}} = \frac{r_{b1}}{r_{b2}} = u_{12},$$

so the transmission ratio is uniquely defined by the ratio of base circle radiuses. It means that when we change center distance a_w, and thus, pitch circle radiuses r_{w1} and r_{w2} as well as pressure angle α_w, transmission ratio u_{12} does not change.

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[1] *Collins English Dictionary. Copyright © HarperCollins Publishers*

9.2.2. Configuration of involute gearing

In gears similar teeth profiles are placed on the pitch distance along base pitch p_b (Fig. 9.7) and, according to general features of involute, they are equidistant (see Fig. 9.5).

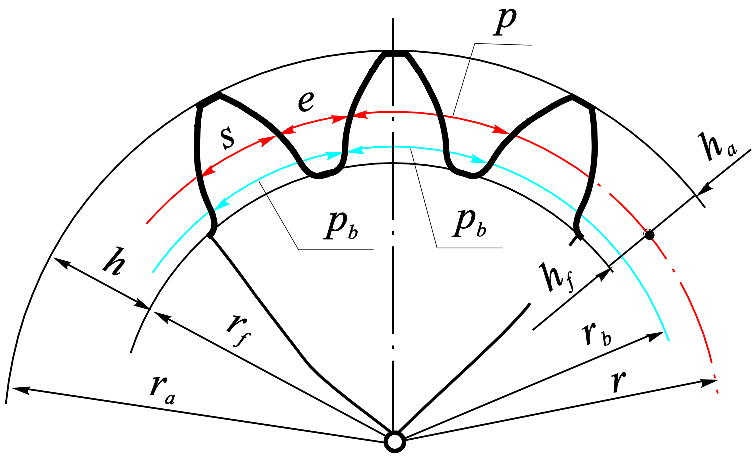


Fig. 9.7. Involute gear configuration

Circular pitch is distance by circular arc of a set radius between similar profiles of two adjacent profiles.

If number of teeth *z*, so pitch along base circle – *base pitch*:

$$p_b = \frac{2\pi r_b}{z}. \tag{9.1}$$

According to (9.1) pitch along circular arc of a random radius $p = \frac{2\pi r}{z}$. Hence

$$r = \frac{p}{2\pi} z.$$

Generally r – is the irrational number, which greatly complicates measurement and control of this magnitude. Let us mark

$$\frac{p}{\pi} = m.$$

Here m – *metric module* (forth *module*), which is taken as a rational number. Module is measured in millimeters and is chosen from a standard set according to Metric Gearing Standards ISO 54:1996 “Cylindrical gears for general engineering and for heavy engineering – Modules” or ДСТУ ISO 54-2001 “Передачі зубчасті циліндричні для загального і важкого машинобудування. Модулі” (See Appendix 2).

Circle, for which module – is rational (standard) number is called standard or nominal pitch circle. This circle divides a tooth into the point of a tooth (gear tip, addendum) and the root of a tooth (dedendum).

In Fig. 9.7 h_a is *tooth addendum*, and h_f is *tooth dedendum*. Here also are: s – *circular tooth thickness*; e – *circular notch width*.

Standard pitch circles of gears, which are in toothing, in separate cases can pass through a pitch point so to say coincide with pitch circles. But we should consider that they belong to concrete gears, and pitch circles appear only in engagement.

Circle of the radius r_f (See Fig. 9.7) is called *dedendum* or *root circle*, and of radius r_a – *addendum or outside circle*.

9.2.3. Methods of gear manufacturing

Today the basic method of gear manufacturing is gear cutting.

The majority of modern gearing was produced and improved on the basis of development of their production by cutter. In order to get involute teeth profiles we use two methods: *forming process* and *generating process*.

We use special gear-cutting machines as the equipment.

Forming process or *form-cutting method* consists in forming teeth surfaces with the help of special end-mill type (Fig. 9.8, *a*) or disk-type gear cutters (Fig. 9.8, *b*).

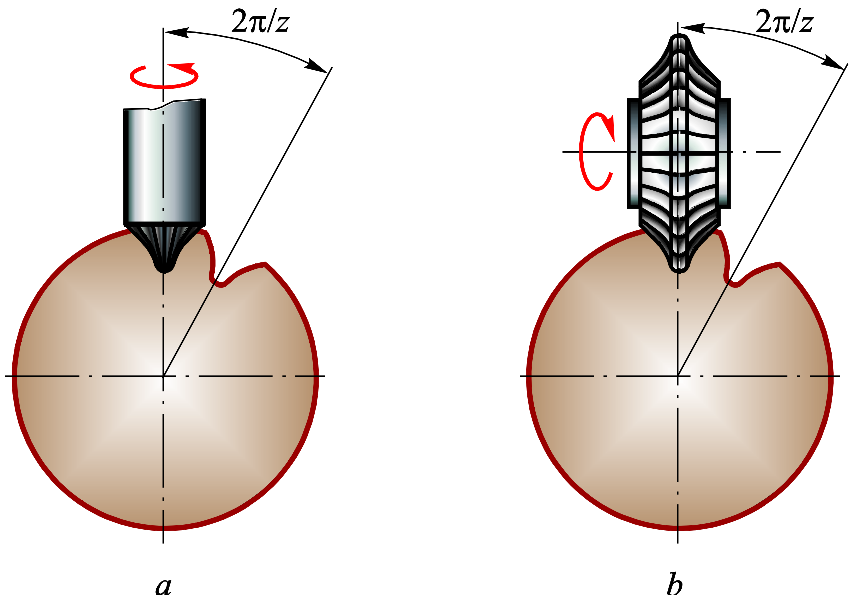


Fig. 9.8. Forming process: *a* – with the help of an end-mill type gear cutter; *b* – with the help of disk-type gear cutter

The contour of cutting part of a tool coincides with the contour of notch between the teeth. When turning, milling cutter displaces along generating line of a tooth, and as a result, one notch is formed. After this milling cutter returns to the initial position, and gear blank turns on the angle $2\pi/z$, where z – is number of teeth of a cut gear, and the process repeats.

This method is rarely used, because it needs numerous toolkits for gear cutting. Moreover, in comparison with other methods it is less productive and not accurate enough.

Generating process was theoretically justified by French geometrician Theodore Olivier, who offered two variants of this method:

1) both mating teeth surfaces of gears are cut by one *generating surface* (see below), which differs from required mating surfaces;
2) generating surface coincides with one of required mating surfaces, relative motion of a generating surface and a gear blank should be the same as of required mating surfaces.

Theodore Olivier (1793–1853)

T. Olivier was a French mathematician and mechanic, one of the founders of the theory of gearing, developed a geometric theory of gearing, and proposed the envelope method as the main way of obtaining any gears. He was a military engineer, taught in the Artillery School at Metz, was one of the organizers of the Polytechnic School in Sweden, and later he was one of the founder of the Central School of Arts and Manufactures in Paris. His main scientific research concerned descriptive and differential geometry.

First variant of methods, offered by Olivier, is shown on the example of gear cutting by a tool, which is made either as generating gear with noses on teeth (shaping cutter), or as a rack, which can be considered as limiting form of gears, when number of teeth tends to infinity. For the rack all circles become straights, and involute tooth profile – straight, which forms angle α with a perpendicular to these straights (Fig. 9.9).

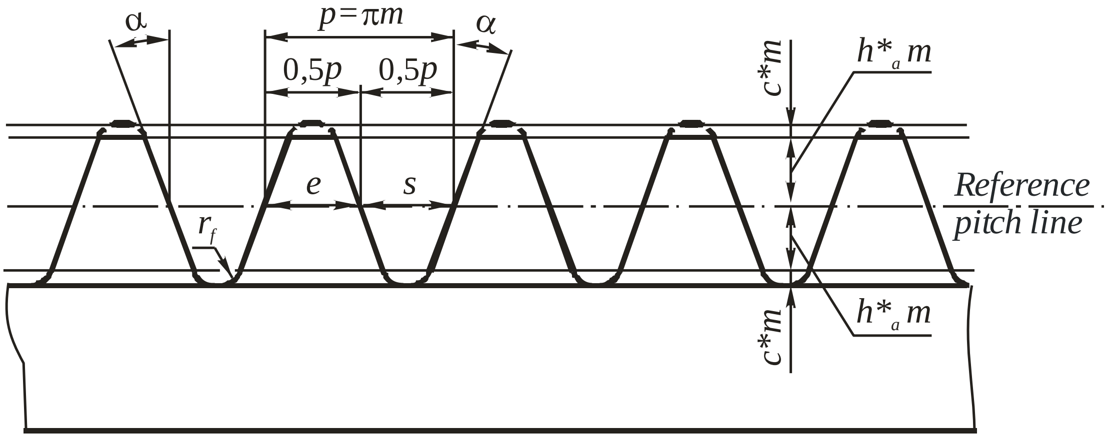


Fig. 9.9. Basic rack

Today gear-cutting hob is often used instead of rack (Fig. 9.10). It is a screw with noses on teeth. If we make axial section of a hob, we get a rack.

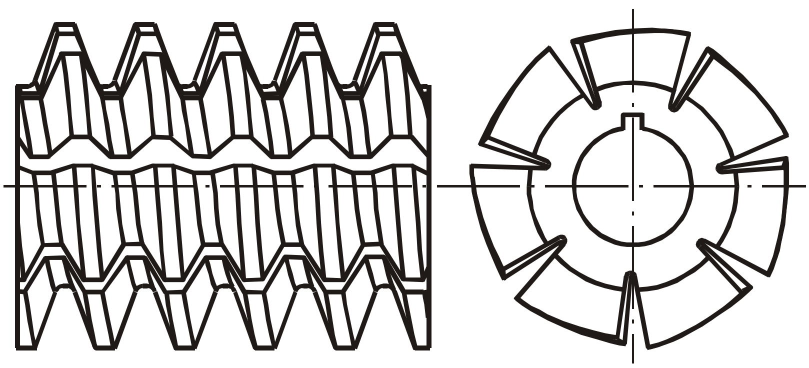


Fig. 9.10 Gear-cutting hob

For a single-thread hob worm angle is not more than 5º. Shaping cutters are used, as a rule, when cutting internal teeth (Fig. 9.11, *a*).

There are three main motions of a gear-cutting tool and gear blank in the process of generating cutting of gears (Fig. 9.11):

– *cutting motion* (parallel motion for shaping cutters or turning motion for a hob) – is made by the gear-cutting tool relatively to the housing and stationary gear blank;

– *generating motion* – is the *motion*, which is made by the gear-cutting tool relatively to the gear blank;

– *feed motion* is made by the gear-cutting tool relatively to the gear blank in the direction to its center.

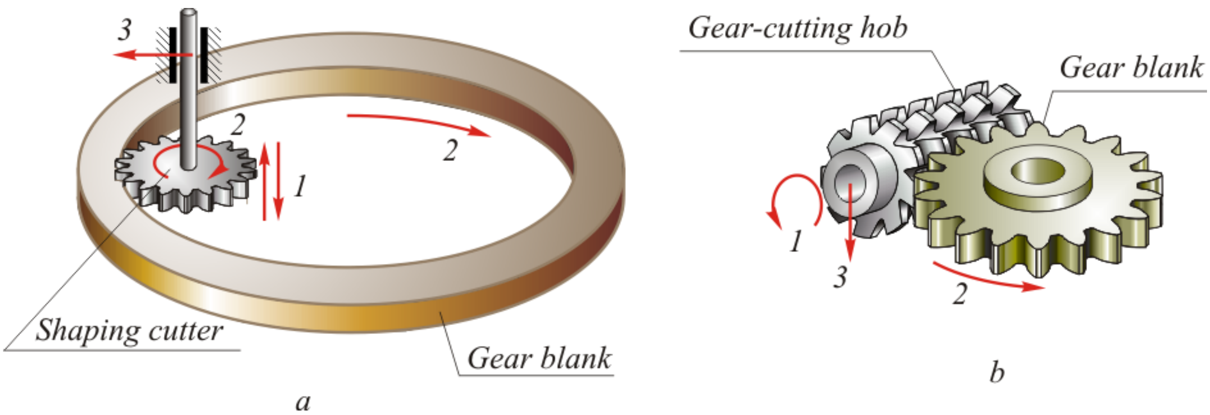


Fig. 9.11 Main motions of a gear-cutting tool and gear blank in the generating cutting process: *a* – with using the shaping cutter; *b* – with using the gear-cutting hob

In one cutting motion the gear-cutting tool forms a so-called *generating surface* on a gear blank.

During generating motion the gear-cutting tool and gear blank reproduce relative motion, which could belong to two gears, which are in true engagement. That is why the tool is made in the form of gear (shaping cutter) or rack.

Feed motion is present in the process of cutting for reduction of cutting forces.

In order to understand, how the forming of teeth surfaces is carried out under generating cutting of gears, let us study the interaction of a gear blank with a rack-shaped cutter.

As we see in Fig. 9.9, rack profile consists of straight and curvilinear parts. Theoretical tooth profile of a rack should be straight. In reality tooth addendum of a rack is extended by the value of *radial (top) clearance* (hatch in figure). The point of tooth of cutter forms gear root, and at tooth action between point of tooth of one gear and bottom land of another gear (in other words – between addendum circle of one gear and root circle of another one) radial clearance should be guaranteed.

Let us suppose that within one cutting motion the gear-cutting tool enters by full depth into the gear blank (feed motion is left out, though in practice it is impossible). Rack tooth forms generating surface on the gear blank. Its straight part forms plane, and curvilinear part – some radial surface.

Generating motion is followed by first cutting motion, so to say turning of a gear blank and parallel motion of a rack that imitates relative motion in the process of engagement of a gear and a rack. Further we have second cutting motion. On the blank the second generating surface is formed etc. Within a turning of a gear blank by one pitch a surface of one tooth, which is inside envelope surface of all generating surfaces, is formed (Fig. 9.12).

Rack contour, which forms toothed surface on the gear blank, is called *basic rack tooth profile*. It assigns form and dimensions of cut teeth.

Parameters of *basic rack* are standardized. According to ISO 53:1998 “Cylindrical gears for general and heavy engineering – Standard basic rack tooth profile” (DSTU ISO 53-2001) they have such value:

- pressure angle $\alpha = 20°$;
- addendum $h_a = m$ ($h_a = h_a^* m$, where $h_a^* = 1$ – coefficient of tooth addendum);
- dedendum $h_f \geq 1.25m$;
- whole depth $h \geq 2.25m$;
- radial (top) clearance $c = 0,25m$ (coefficient of radial clearance $c^* = 0,25$);

We should mention that straight part of a basic rack tooth profile forms involute part of a tooth, and rounded – fillet surface (transition curve from involute to the root circle)

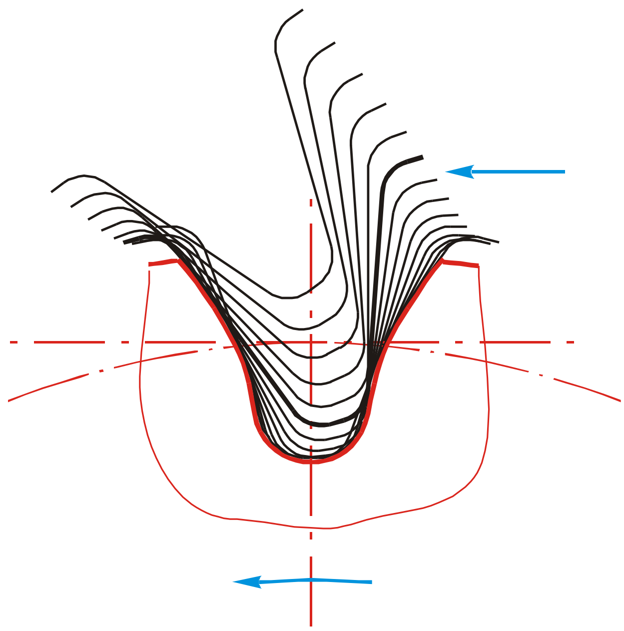


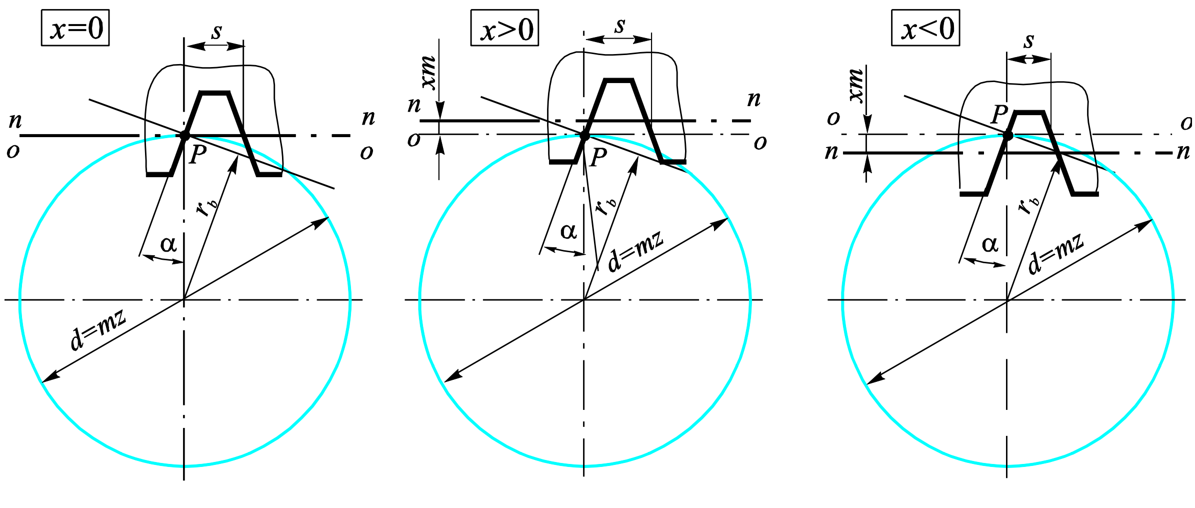
Fig. 9.12. Generating surfaces on the blank

Depending on basic rack and gear blank positional relationship, in the finishing moment of gear-cutting process there are three variants of gear cutting (Fig. 9.13).

а) reference pitch line of a rack tangents to standard pitch circle of a gear (Fig. 9.13, *a*);

б) reference pitch line of a rack doesn't tangent to standard pitch circle of a gear (Fig. 9.13, *b*);

в) reference pitch line of a rack intersects standard pitch circle of a gear (Fig. 9.13, *c*);



a *b* *c*

Fig. 9.13. Basic rack and gear blank positional relationships:
a – the zero placing of a tool; *b* – the positive placing of a tool;
c – the negative placing of a tool

In the first variant there is zero placing of a gear-cutting tool, and the gear is told to be cut without shifting:

$$\chi = xm = 0; \;\; x = 0.$$

Here χ –is a magnitude, multiple by module, and is called *a shift*, mm; *x – is shift factor or correction factor for profile shift*.

Circular tooth thickness is equal to circular notch width of a rack by reference pitch line:

$$s = 0{,}5\pi m.$$

In the second variant there is positive placing of a tool, and the gear is told to be cut with positive shifting:

$$\chi = xm > 0; \qquad x > 0.$$

Circular tooth thickness is more than circular notch width of a rack by reference pitch line. According to Fig. 9.13, *b*

$$s = 0{,}5\pi m + 2xm\ \mathrm{tg}\alpha.$$

Thus circular tooth thickness of a gear, cut with positive shift is more than of shiftless gear. It is also evident that the gear cut with positive shift, has bigger circular tooth thickness *s* than circular notch width of a rack *e* (See Fig. 9.7).

In the third variant there is negative placing of a gear-cutting tool, and the gear is told to be cut with negative shifting.

$$\chi = xm < 0; \qquad x < 0.$$

Circular tooth thickness of a gear is less than circular notch width of a rack by reference pitch line. According to Fig. 9.13, *c*

$$s = 0{,}5\pi m - 2xm\ \mathrm{tg}\alpha. \tag{9.2}$$

Thus, circular tooth thickness of a gear, cut with negative shift is less than of shiftless gear, and correspondingly, circular tooth thickness is less than circular notch width of a rack.

Independently of shift, gears are cut by one basic rack of a set module with any number of teeth have mating surfaces, so to say they form an accurate flank clearance-free toothing. Base radiuses, as it was already mentioned, does not change. According to Fig. 9.13, they are related with nominal pitch radiuses by the ratio:

$$r_b = r\cos\alpha \quad \text{or} \quad r_b = 0{,}5mz\cos\alpha .$$

Hence we may conclude that shift affects only circular tooth thickness of a gear and placement of a used involute section.

9.2.4. Determination of tooth dimensions

There are three types of gearing depending on shifts obtained when cutting gears (Fig. 9.14). They differ in positional relationship of standard pitch circles and *working pitch circles* (centrodes).

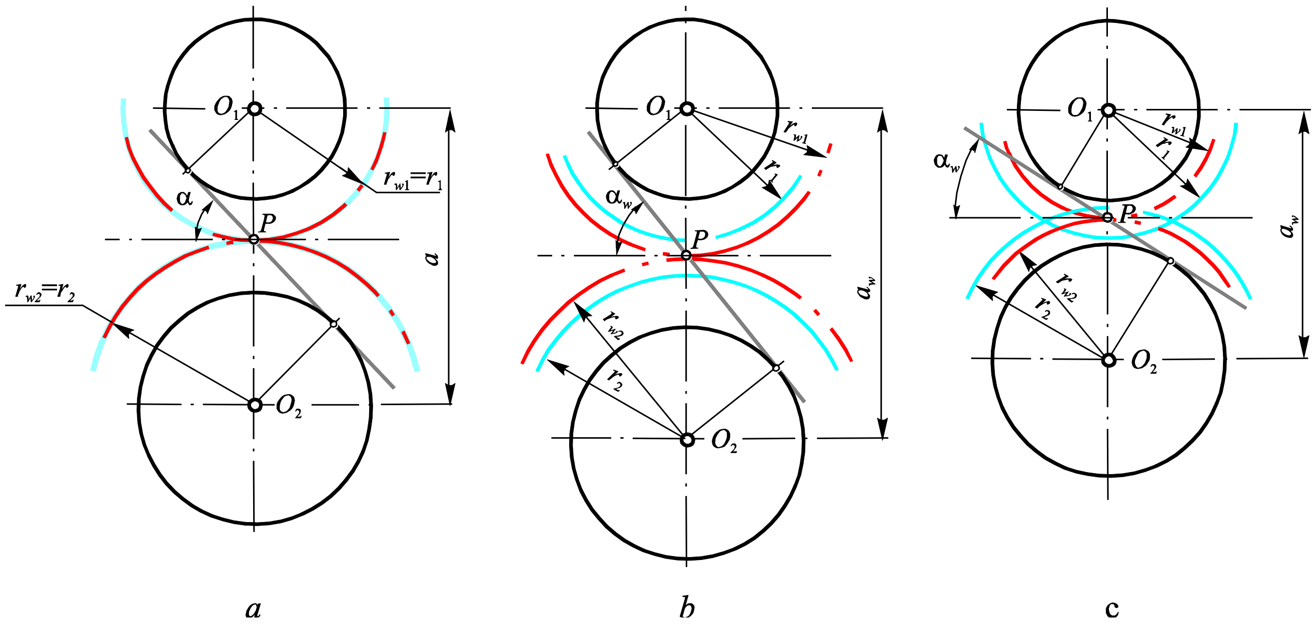


Fig. 9.14. Positional relationship of working and standard pitch circles: $a - x_1 = x_2 = 0$ or $|x_1| = |-x_2|$; $b - x_1 + x_2 > 0$; $c - x_1 + x_2 < 0$

1. Working and standard pitch circles of gears coincide (Fig. 9.14, *a*):

$$r_1 = r_{w1}; \quad r_2 = r_{w2}; \; \alpha_w = \alpha = 20°;$$

$$a_w = a = \frac{d_1 + d_2}{2} = r_1 + r_2.$$

This happens when circular tooth thickness of the first gear coincides with circular notch width of the second gear (for pitch circles these dimensions of gears always coincide). This condition can be realized, when gears are either shiftless, or shifted, equal by absolute value, opposite in sign:

$$x_1 = x_2 = 0 \quad \text{or} \quad |x_1| = |-x_2|.$$

2. Circular tooth thickness of the first gear is bigger than circular notch width of the second gear.

In such case working and standard pitch circles of gears do not coincide (Fig. 9.14, *b*). Center distance a_w will be bigger than the sum of standard pitch radiuses:

$$a_w > \frac{d_1 + d_2}{2}.$$

Pressure angle also increases:

$$\alpha_w > \alpha = 20°.$$

Such type of gearing can be obtained under the condition

$$x_1 + x_2 > 0.$$

3. Circular tooth thickness of the first gear is less than circular notch width of the second gear.

As in a previous case, here working and standard pitch circles of gears do not coincide (Fig. 9.14, *c*). But center distance a_w will be less than the sum of standard pitch radiuses:

$$a_w < \frac{d_1 + d_2}{2}.$$

Pressure angle decreases:

$$\alpha_w < \alpha = 20°.$$

Such type of gearing can be obtained under the condition

$$x_1 + x_2 < 0.$$

Thus, we should find a_w and α_w, other dimensions of the gearing and its elements. For this reason we should study some features of involute.

Let us study abstract point *M* of the involute (Fig. 9.15). Angle ν_M between radiuses *OC* and *OB*, built to a limit point *C* of the involute and in the tangent point *B* of generating line *q* to base circle, is called *sweeping angle of involute* in the point *M*. In the point *C* $\nu_C = 0$.

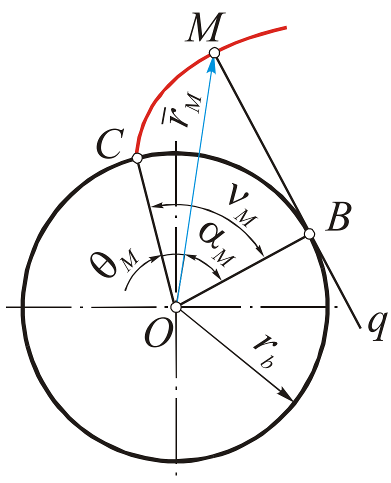


Fig. 9.15 Involute parameters

Angle α_M between radiuses *OB* and *OM* is called profile angle in the point *M*.

From a right-angled triangle *OBM*

$$\alpha_M = \arccos(r_b / r_M),$$

where $r_M = |\vec{r}_M|$ – is magnitude of the radius-vectorof the point *M* of the involute.

Angle

$$\theta_M = \nu_M - \alpha_M$$

is called *involute angle*.

Considering features of involute, we have $\cup CB = MB$. Then from the ΔOBM we have

$$\mathrm{tg}\alpha_M = \frac{MB}{OB} = \frac{\cup CB}{OB} = \frac{r_b \nu_M}{r_b} = \nu_M = \theta_M + \alpha_M .$$

Hence

$$\theta_M = \mathrm{tg}\alpha_M - \alpha_M = \mathrm{inv}\alpha_M .$$

Relationship $\theta_M = \mathrm{inv}\alpha_M$ is called *involute function*. With its help we can define basic dimensions of gearing and its elements.

Linking criterion of two gears is absence of backlashes in toothing – so to say creation of *accurate toothing*. In analytical form this criterion is:

$$p_w = s_{w1} + s_{w2}, \tag{9.3}$$

where p_w – is a pitch along pitch circular:

$$p_w = \frac{2\pi r_{w1}}{z_1} = \frac{2\pi r_{w2}}{z_2}; \tag{9.4}$$

s_{w1} and s_{w2} are circular tooth thicknesses along pitch circles of the first and second gears respectively.

Tooth thicknesses along standard pitch circles are defined by formula (9.2). Marking in this equation $2x\mathrm{tg}\alpha = \delta$, we may write down the following equations for the first and second gears relatively:

$$s_1 = m\left(\frac{\pi}{2} + \delta_1\right); \qquad s_2 = m\left(\frac{\pi}{2} + \delta_2\right). \tag{9.5}$$

Members δ_1 and δ_2 characterize change in tooth thicknesses along standard pitch circles as a result of tool shifting (See Fig. 9.13).

In order to define real pressure angle α_M we use a basic property of involute (Fig. 9.16):

$$\psi_i + \theta_i = \psi + \theta. \tag{9.6}$$

Here θ_i, θ – are involute angles:

$$\theta_i = \text{inv}\alpha_i; \;\; \theta = \text{inv}\alpha;$$

ψ_i, ψ – are central angles:

$$\psi_i = \frac{s_i}{2r_i}; \qquad \psi = \frac{s}{2r}; \tag{9.7}$$

s_i, s – are tooth thicknesses along random (radius r_i) and standard pitch circles respectively.

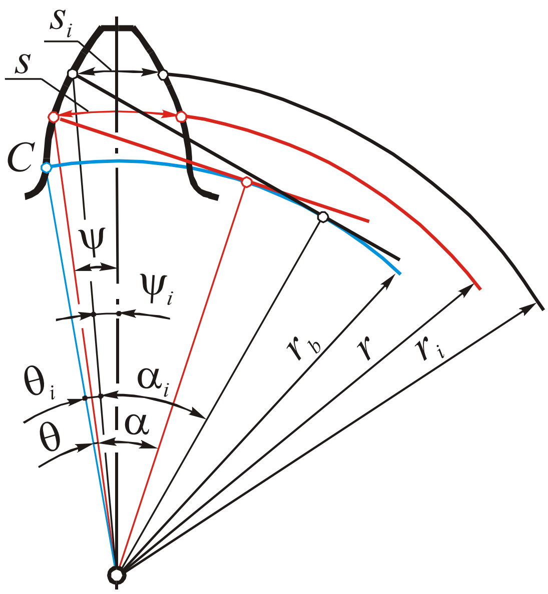


Fig. 9.16. Involute tooth profile

After substitution into (9.6) we have:

$$s_i = s\left(r_i / r\right) - 2r_i\left(\text{inv}\alpha_i - \text{inv}\alpha\right). \tag{9.8}$$

Taking into account (9.5) and (9.7) and the fact that standard pitch radius is defined by the formula $r = 0{,}5mz$, according to (9.8) tooth thicknesses along working pitch circles:

$$s_{w1} = s_1\left(r_{w1}/r_1\right) - 2r_{w1}\left(\theta_w - \theta_1\right) = r_{w1}\left[\frac{\pi}{z_1} + 2\left(\frac{\delta_1}{z_1}\right) - 2\left(\theta_w - \theta_1\right)\right] = \frac{p_w}{2}; \qquad (9.9)$$

$$s_{w2} = s_2\left(r_{w2}/r_2\right) - 2r_{w2}\left(\theta_w - \theta_2\right) = r_{w2}\left[\frac{\pi}{z_2} + 2\left(\frac{\delta_2}{z_2}\right) - 2\left(\theta_w - \theta_2\right)\right] = \frac{p_w}{2}. \qquad (9.10)$$

Here $\theta_{w1} = \theta_{w2} = \theta_w = \mathrm{inv}\alpha_w$.

Let's put (9.9) and (9.10) into (9.3). Considering (9.4) we obtain:

$$r_{w1}\left[\frac{\pi}{z_1} + 2\left(\frac{\delta_1}{z_1}\right) - 2\left(\theta_w - \theta_1\right)\right] + r_{w2}\left[\frac{\pi}{z_2} + 2\left(\frac{\delta_2}{z_2}\right) - 2\left(\theta_w - \theta_2\right)\right] = \frac{2\pi r_{w1}}{z_1} = \frac{2\pi r_{w2}}{z_2}. \quad (9.11)$$

Transmission ratio of cylindrical gearing is $i_{12} = \frac{r_{w2}}{r_{w1}} = \frac{z_2}{z_1}$. Hence we obtain ratio $r_{w1} = r_{w2}\left(z_1/z_2\right)$ or $r_{w2} = r_{w1}\left(z_2/z_1\right)$. After having substituted one of them into (9.11) and required transformations we obtain:

$$2\pi + 2\left(\delta_1 + \delta_2\right) - 2\left(z_1 + z_2\right)\mathrm{inv}\alpha_w + 2\left(z_1 + z_2\right)\mathrm{inv}\alpha = 2\pi.$$

Hence, considering that $\delta_1 = 2x_1\mathrm{tg}\alpha$ and $\delta_2 = 2x_2\mathrm{tg}\alpha$, we may define involute of the pressure angle of designed gearing α_w which is called *working pressure angle*:

$$\mathrm{inv}\alpha_w = \mathrm{inv}\alpha + 2\frac{x_1 + x_2}{z_1 + z_2}\mathrm{tg}\alpha.$$

Here $\mathrm{inv}\alpha = \mathrm{tg}\alpha - \alpha$ ($\alpha = 20^\circ$ according to ISO 53:1998 (DSTU ISO 53-2001)).

When we have found $\mathrm{inv}\alpha_w$, we use tables of involutes to define value α_w, which enables us to calculate all required dimensions of the gearing.

1. Standard pitch radiuses:

$$r_1 = \frac{mz_1}{2}; \qquad r_2 = \frac{mz_2}{2}.$$

2. Base radiuses:

$$r_{b1} = r_1 \cos\alpha; \qquad r_{b2} = r_2 \cos\alpha.$$

3. Working pitch radiuses:

$$r_{w1} = \frac{mz_1}{2}\frac{\cos\alpha}{\cos\alpha_W}; \qquad r_{w2} = \frac{mz_2}{2}\frac{\cos\alpha}{\cos\alpha_W}.$$

4. Center distance:

$$a_w = \frac{m\left(z_1 + z_2\right)\cos\alpha}{2\cos\alpha_w}. \tag{9.12}$$

Equation (9.12) may also be written as

$$a_w = a + ym, \tag{9.13}$$

where y – is *coefficient of effective addendum modification* aka *centre distance increment factor*. From (9.13) we get

$$y = \frac{z_1 + z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right).$$

5. Root radiuses:

$$r_{f1} = r_1 - m\left(h_a^* + c^* - x_1\right); \quad r_{f2} = r_2 - m\left(h_a^* + c^* - x_2\right). \tag{9.14}$$

6. Outside radiuses:

$$r_{a1} = a_w - r_{f2} - c^* m \qquad r_{a2} = a_w - r_{f1} - c^* m. \tag{9.15}$$

These radiuses may be defined through correcting value Δy:

$$\Delta y = x_1 + x_2 - y. \tag{9.16}$$

After having made some easy transformations the formulas (9.15) look like:

$$r_{a1} = r_1 + m\left(h_a^* + x_1 - \Delta y\right); \quad r_{a2} = r_2 + m\left(h_a^* + x_2 - \Delta y\right). \tag{9.17}$$

Let's memorize that according to ISO 53:1998 (DSTU ISO 53-2001) $h_a^* = 1$, $c^* = 0{,}25$.

If gears are cut without shift ($x_1 = x_2 = 0$ and $\alpha_w = \alpha$), formulas for determination of basic dimensions of a gearing are simplified:

1. Standard and working pitch radiuses

$$r = r_w = \frac{mz}{2}.$$

2. Center distance

$$a_w = a = \frac{m}{2}\left(z_1 + z_2\right).$$

3. Root radiuses

$$r_f = \frac{m}{2}\left(z - 2{,}5\right).$$

4. Outside radiuses

$$r_a = \frac{m}{2}\left(z + 2\right).$$

9.2.5. Helical gearing

According to tooth forms there are *spur* and *helical* gears.

For spur tooth surface is made by parallel motion of involute in direction parallel to axis of gear rotation. If standard pitch cylinder of a gear is developed on a plane

(Fig. 9.17, *a*), intersections of tooth surfaces and standard pitch cylinder are shown by straight lines, parallel to axis of gear rotation.

In helical gears tooth surface is formed as a result of helical motion of involute by standard pitch cylinder. If we develop on a plane the standard pitch cylinder of such gear (Fig. 9.17, *b*), helical intersections of this cylinder with tooth surfaces of gears will be shown by straight lines, lopsided by the angle β to the axis of gear rotation.

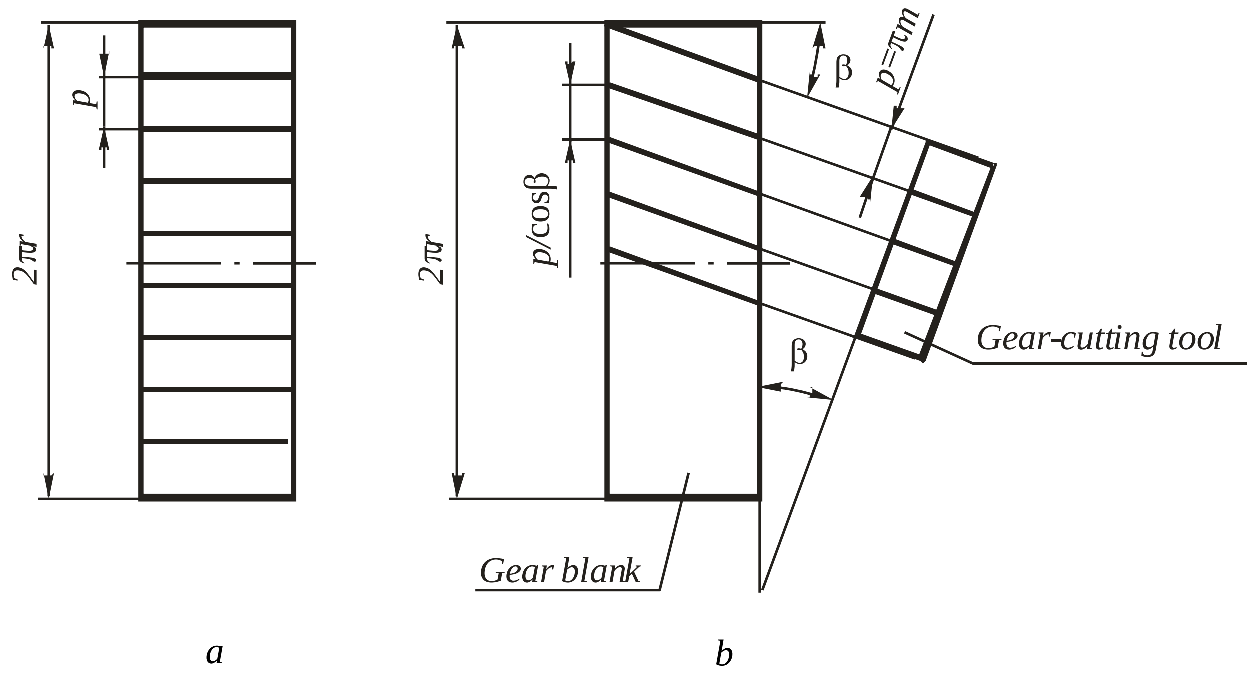


Fig. 9.17. Spur (*a*) and helical (*b*) gears

Helical gears are cut by the same tool as spur gears, but it is installed obliquely under $\angle\beta$ to gear face *t–t* (See Fig. 9.17, *b*). As a result of obliquity *circular pitch in plane of rotation* aka *transverse circular pitch* increases:

$$p_t = \frac{p}{\cos\beta},$$

it means that module in face section increases in comparison with standard module *m* (in this case it is called *normal module* because measured in a plane perpendicular to the teeth):

$$m_t = \frac{m}{\cos\beta}.$$

The module m_t is called *transverse module.*

When defining dimensions of helical gears in calculating formulas we should put transverse module of the gear instead of standard module *m*. For example, standard pitch radius

$$r = \frac{m_t z}{2} = \frac{mz}{2\cos\beta}.$$

We should admit that dimensions of helical tooth in height do not change in comparison with spur ones, as we use the same instrument for cutting. So when finding dimensions of helical gears parameters of basic rack tooth profile h_a^*, c^* as well as Δy and x in formulas (9.14) – (9.17) should be multiplied not by transverse module, but by normal module. For example:

$$r_f = \frac{mz}{2\cos\beta} - m\left(h_a^* + c^* - x\right).$$

Due to turning of tool the pressure angle in helical gear increases in comparison with standard value $\alpha = 20°$. It is called *transverse pressure angle* and defined by formula:

$$\text{tg}\alpha_t = \frac{\text{tg}\alpha}{\cos\beta}.$$

9.2.6. External involute toothing elements

Let’s construct an image of toothing, so to say we show gear teeth, which are in engagement (Fig. 9.17). For this reason we should calculate all required gear dimensions and define coordinates of points of tooth profiles (methodology of such construction is studied in term project and is described in details in [5]).

Let’s study toothing elements, shown in Fig. 9.18.

1. *Theoretical line of action* or *pressure line* – is a line N_1N_2, which is common tangent to base circle (See item 9.2.1).

2. *Effective line of action* or *active portion of the line of action* $B_{p1}B_{p2}$.

Effective line of action – is locus of contact points of one pair of teeth from the beginning till the end of a gearing.

In points B_{p1} and B_{p2} *theoretical line of action* intersects with outside circle of gears. In one of these points depending on the turning direction, for example in a point B_{p1}, mating profiles mesh, and in another – B_{p2}, they become disconnected.

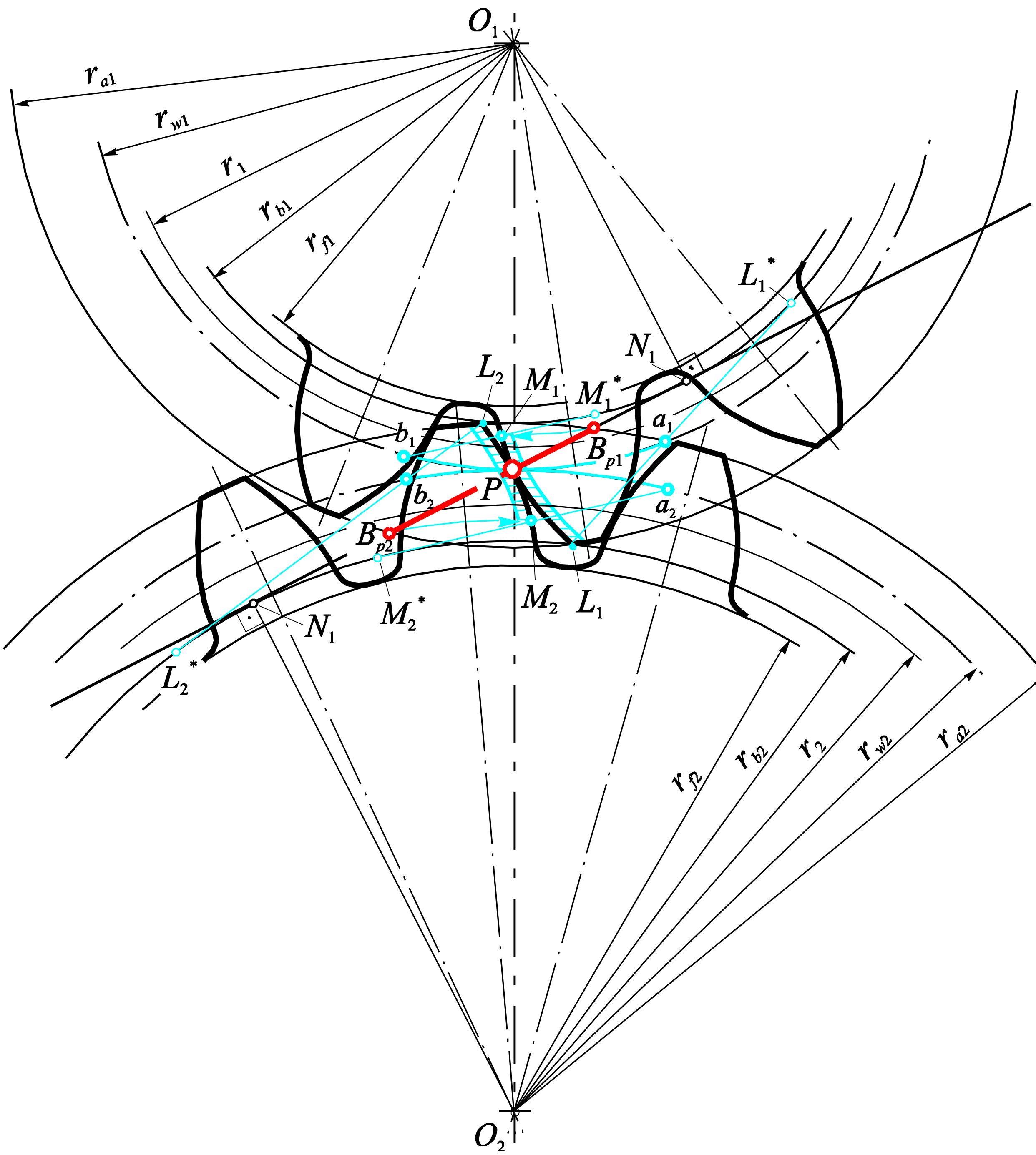


Fig. 9.18. Toothing elements

3. *Active gear tooth flanks or working flanks.*

Active gear tooth flanks – are segments of mating profiles, points of which interact in toothing.

In the Fig. 9.17 they are shaded. On gear tips working flanks are limited by outside circles, so to say by points L_1 and L_2, and on roots – by points M_1 and M_2, mated with them. Points L_1 and M_2; L_2 and M_1 meet on the line of action in points B_{p2} and B_{p1} in the first or last moment of profiles' contact respectively. In order to find the point M_2 on the root of the second gear, we should build an arc by the radius O_2B_{p2} to the intersection with the tooth profile. In the same way we find the point M_1 on the root of the second gear (See Fig. 9.17).

4. *Arc of contact.*

Arc of contact – is a segment of a working pitch circle, which corresponds to the rotation angle of a gear per time of tooth pair engagement.

In order to find an arc of contact, for example for the second gear, we should build tangents to a base circle from the points, which limit working flanks of gear (in Fig. 9.17 these are lines $L_2L_2^*$ and $M_2M_2^*$). Their crossing points with a working pitch circle will form the arc of contact $\cup a_2b_2$. In the same way we define the arc of contact for the first gear $\cup a_1b_1$.

As working pitch circles in toothing roll without sliding, so the following condition should be fulfilled:

$$\cup a_1b_1 = \cup a_2b_2 .$$

9.2.7. Effect of gear-cutting tool shift on tooth forms during cutting

Fig. 9.19 shows tooth forms of three gears with equal number of teeth, cut by one gear-cutting tool, but with different shifts.

From the figure we see that magnitudes of shift factors of gears are interdependent as:

$$x_3 > x_2 > x_1 .$$

As gears have equal standard pitch radiuses, so circular tooth thickness increases with the increase of shift magnitude, so to say $s_3 > s_2 > s_1$. Here $s_3 = \cup ad;\ s_2 = \cup ac;\ s_1 = \cup ab$. Root radiuses and outside radiuses also increase. Tooth roots become thicker, but their tooth points become thinner.

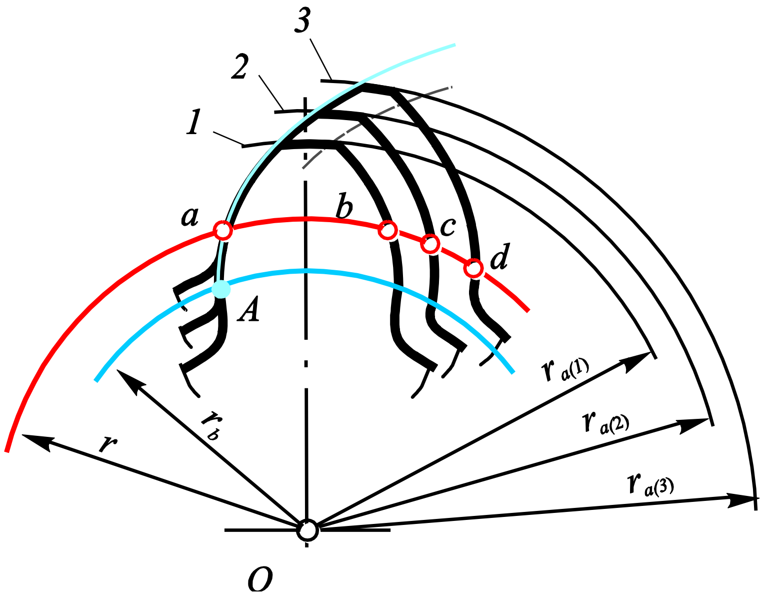


Fig. 9.19. Tooth forms depended on shift magnitude

If we consider a tooth as a cantilever bar, loaded with concentrated force, the more the shift is, that is the thicker tooth root is, the more bending strength it has.

On the other hand, as base circles of gears are also equal between themselves, so tooth profiles are outlined by the same involute. But depending on the shift magnitude, congruous profiles are outlined by different segments of involute. According to Fig. 9.19, the more the shift is, the farther from a base (point A) segment of the involute outlines the tooth profile. Radius of profile curvature increases. Contact stresses, according to the Hertz formula, decreases, which contributes to the decrease of surface deterioration.

Thus, choosing a shift factor when designing a gearing, we may affect tooth form, as well as toothing quality.

9.2.8. Gearing quality indicators

For the gearing quality rating we introduce *gearing quality indicators*. They help to estimate the gearing from the point of view of a silent action and its operation smoothness, possible wear and strength of teeth. With their help, optimal gearings are designed by choosing the rational values of shift factors.

Quality indicators of a gearing include:

– *contact ratio*;

– *specific sliding ratio*;

– *specific pressure ratio*.

Contact ratio. In order for the engagement to be continuous and smooth, it is necessary that at the disengagement of the first pair of teeth, at least one more pair be engaged. It is possible under the condition that a length of an arc of contact will be more than a circular pitch (along the working pitch circle).

Fig. 9.20 shows areas, which are locus of contact lines of teeth during a spur gears engagement.

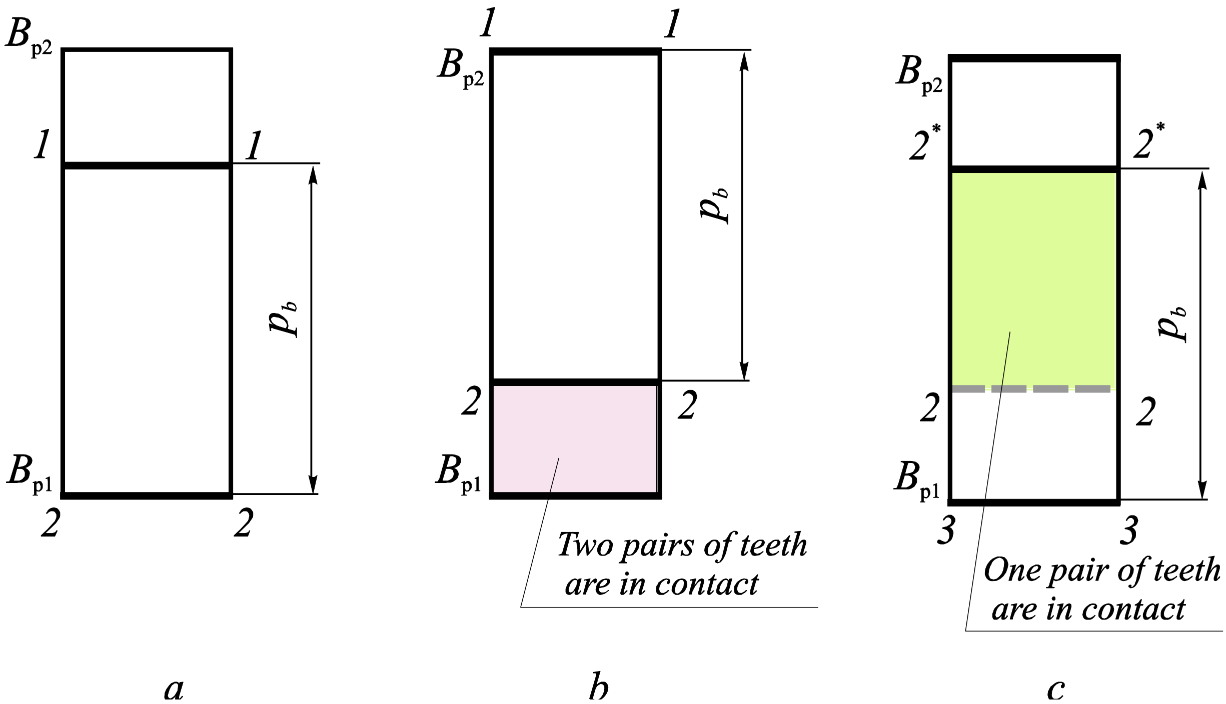


Fig. 9.20. Locus of contact lines of teeth during a spur gears engagement: *a* – the first stage of engagement; *b* – the second stage of engagement; *c* – the third stage of engagement;

Width of an area equals to width of a gear, and height equals to a length of effective line of action $B_{p1}B_{p2}$ (See Fig. 9.18). Distance between two neighboring contact lines in these figures are distances between two involutes along common normal to them, so to say along the line of action. This distance according to features of involute is equal to the distance along base circle between these involutes, so to say it is equal to the base pitch p_b.

In the moment, when the pair 2-2 enters the mesh in the point B_{p1}, the pair 1-1 is on the plane of engagement on the base pitch distance from it (Fig. 9.20, *a*). Till the moment of leaving of the meshing by the pair 1-1 in the point B_{p2} two pairs of teeth are in a mesh simultaneously (Fig. 9.20, *b*). In the course of further turning of gears there is a unit pair in the meshing – 2-2, till it takes the position 2^*-2^*, when the next pair of teeth will mesh (Fig. 9.20, *c*).

In such a way the contacting pairs are overlapping each other, providing continuity of engagement.

Contact ratio of spur gearing is the ratio of the arc of contact length to the pitch along the working pitch circle or a length of effective line of action to the base pitch.

$$\varepsilon_\alpha = \frac{\cup ab}{p_w} = \frac{B_{p1}B_{p2}}{p_b} > 1. \qquad (9.18)$$

Admissible value of contact ratio $[\varepsilon_\alpha]$ is defined by the degree of accuracy of a gearing (See Appendix 3).

We should mention that contact ratio ε_α decreases with the increase of the shift factor x.

In helical gearing meshing duration of one pair of teeth increases, so contact ratio of helical gearing ε_γ is bigger than ε_α.

Let's develop a standard pitch cylinder of a gear on a plane (Fig. 9.21). Helical intersections of this cylinder with tooth surfaces of gears are represented as straight lines tilted by angle β and are situated on the distance of circular pitch in plane of rotation p_t (See item 9.2.5).

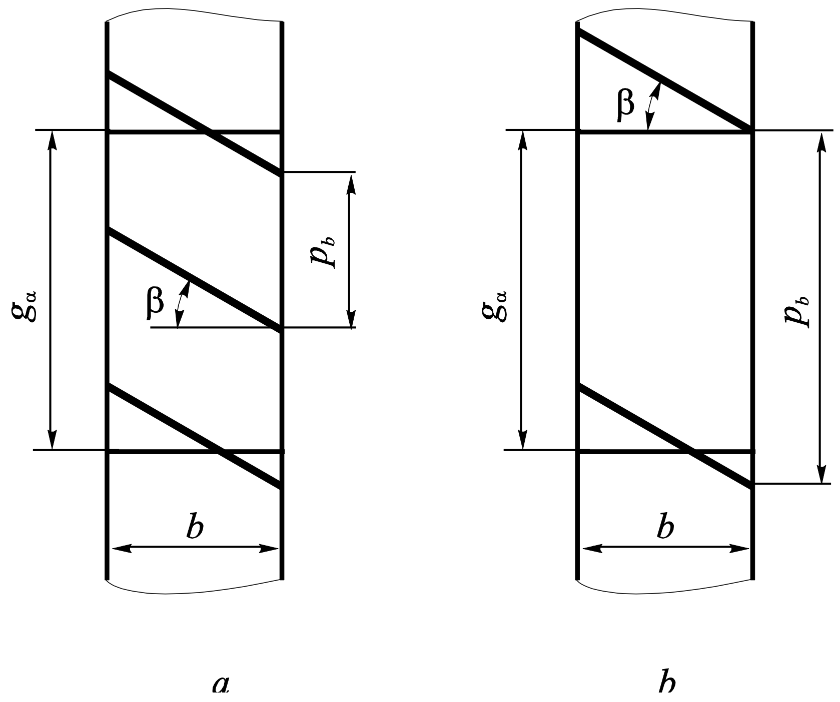


Fig. 9.21. Locus of contact lines of teeth during a helical gears engagement: a – $p_b < g_\alpha$; b – $p_b > g_\alpha$

For the helical gearing the contact ratio is defined as sum of two constituents:

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \, .$$

Here ε_γ – the *total contact ratio* ε_α – the *transverse contact ratio*, which is calculated by the formula (9.18); $\varepsilon_\beta = \dfrac{b\,\mathrm{tg}\beta}{p_t}$ – the *overlap (axial) contact ratio*, where b – is a width of a gear.

Fig. 9.21, *b* shows that even if the length of effective line of action g_α, is less than base pitch p_b, so to say when transverse contact ratio $\varepsilon_\alpha<1$, the engagement stays continuous through a slope of teeth. That is the advantage of the helical gearing.

In practice allowable magnitude of transverse contact ratio in helical gearing is $[\varepsilon_\alpha]=1$.

Specific sliding ratio. It characterizes the slip level of meshed gear teeth in the process of engagement. It is defined as the ratio of slip velocity in contact point K of profiles $\vec{V}_{Slip} = \vec{V}_{K1} - \vec{V}_{K2}$ to the tangential constituent $\vec{V}_K^{\tau}$ of velocity of tooth's point K of a given gear (Fig. 9.22).

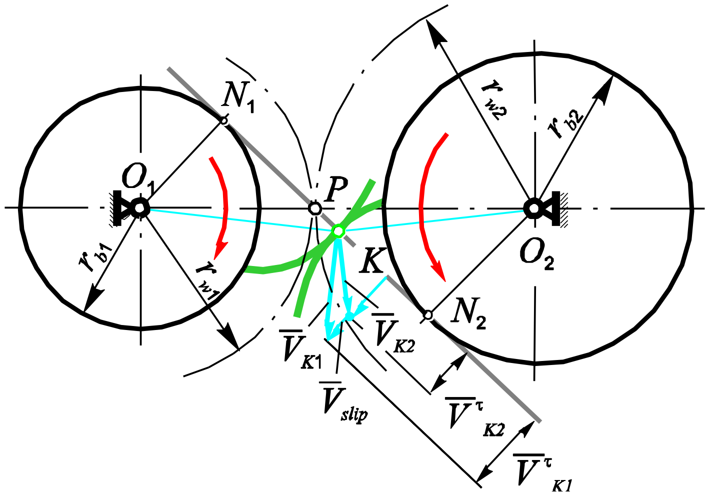


Fig. 9.22. Velocities of gears in a contact point

$$\lambda_1 = \frac{V_{Slip}}{V_{K_1}^{\tau}}; \qquad \lambda_2 = \frac{V_{Slip}}{V_{K_2}^{\tau}}.$$

In order to define specific sliding ratios we may use the following formulas:

$$\begin{cases} \lambda_1 = \left(1 + \dfrac{1}{u_{12}}\right) \dfrac{l_K}{l_K + l_{P1}}; \\ \lambda_2 = \left(1 + \dfrac{1}{u_{12}}\right) \dfrac{l_K}{l_K - l_{P2}}. \end{cases} \qquad (9.19)$$

Here u_{12} is the transmission ratio of gearing; l_K – is the algebraic magnitude, which expresses the distance from the pitch point P to the current position of the contact point K of a pair of teeth; l_{P1} and l_{P2} are modulus of lengths of segments PN_1 and PN_2.

In the process of toothing the contact point of teeth K moves along an effective line of action from the point B_{p1} to the point B_{p2} (See Fig. 9.18). Then the distance l_K will change from ($-B_{p1}P$) to zero, and further from zero to ($+PB_{p2}$).

Fig. 9.23 shows graphs of dependence between specific sliding ratios λ_1 and λ_2 and a position of contact point K on the theoretical line of action N_1N_2, built according to the formulas (9.19).

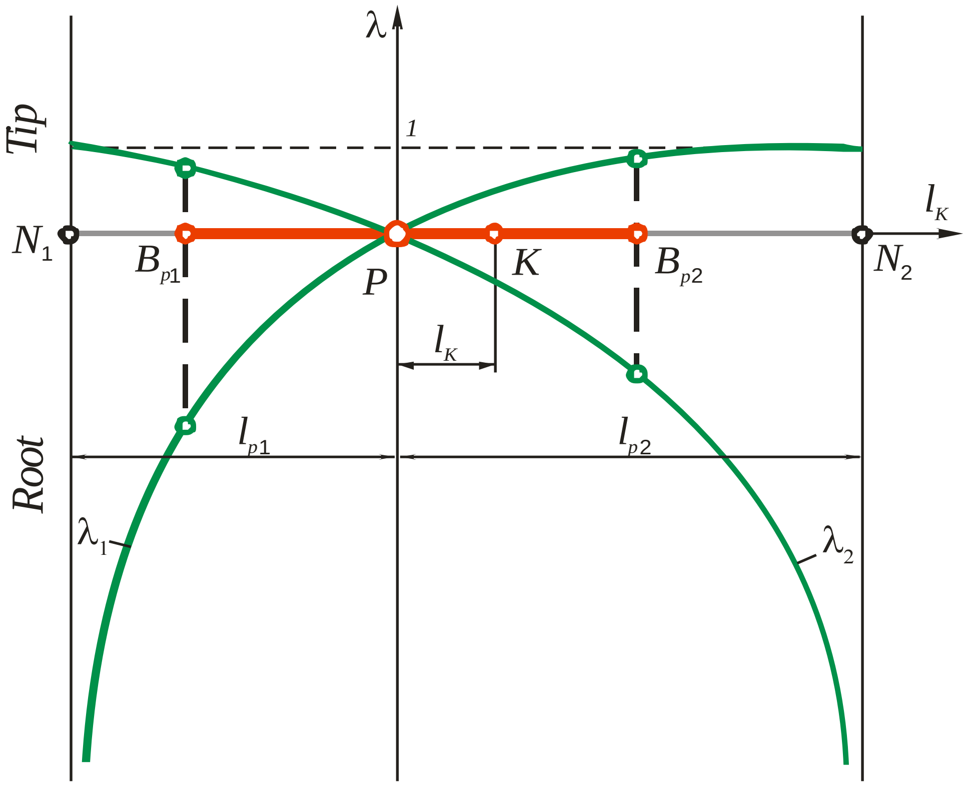


Fig. 9.23. Graphs of dependence between specific sliding ratios λ_1 and λ_2 and a position of contact point K

As we may see the specific sliding ratio λ_1 takes the maximum value on the effective line of action in the point B_{p1}, so to say at the first moment of contact between a pinion root and a gear tip (these are the points M_1 and L_2 of the profiles in Fig. 9.18). Specific sliding ratio λ_2 takes the maximum value in the point B_{p2}, when teeth disengage.

Specific sliding ratios λ_1 and λ_2 depend on shift factors of gears x_1 and x_2.

Specific pressure ratio. Working capacity of a gear is defined by the magnitude of contact stresses at a meshing of a pair of teeth in a contact zone. After these stresses take certain limit values, considering their cycling, acting faces of teeth begin to spall, and the gearing fails. These stresses can be defined by the Hertz formula for the cylinder-to-cylinder contact:

$$\sigma_H = 0{,}418\sqrt{\frac{Q}{b}\frac{E_{зв}}{\rho_{зв}}}.$$

Here Q is the force in contact point; b is the width of a gear (the length of contact line); $E_{rd} = \dfrac{2E_1E_2}{E_1 + E_2}$ is a reduced coefficient of elasticity; $\rho_{rd} = \dfrac{\rho_1\rho_2}{\rho_1 + \rho_2}$ is a reduced radius of curvature, where ρ_1 and ρ_2 are radiuses of involute profile curvatures in contact point (Fig. 9.24).

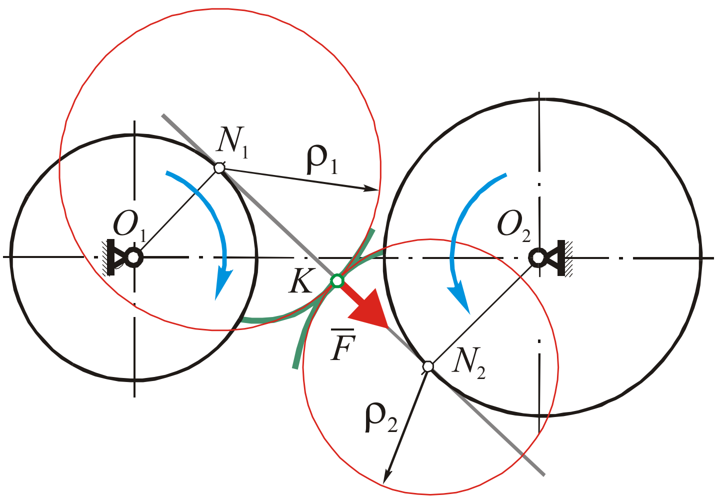


Fig. 9.24. Force in a contact point at a meshing of a pair of teeth

Using symbols in Fig. 9.24, we may write that $\rho_{rd} = \dfrac{N_1K \cdot KN_2}{N_1N_2}$.

Specific pressure ratio takes into account the influence of geometry of teeth (radiuses of their profile curvatures in contact point) on the magnitude of contact stresses. It is defined as ratio

$$\vartheta = \frac{m}{\rho_{rd}} = \frac{m \cdot N_1N_2}{N_1K \cdot KN_2}.$$

Here m – is module of gearing.

Ratio ϑ does not depend on the module magnitude, as the radius of curvature ρ is proportional to module.

As the point K at an engagement of a pair of teeth moves along the line of action, so lengths of segments N_1K and KN_2 change, specific pressure ratio also changes depending on the position of contact point K on the line of action. The graph of such change is represented in Fig. 9.25.

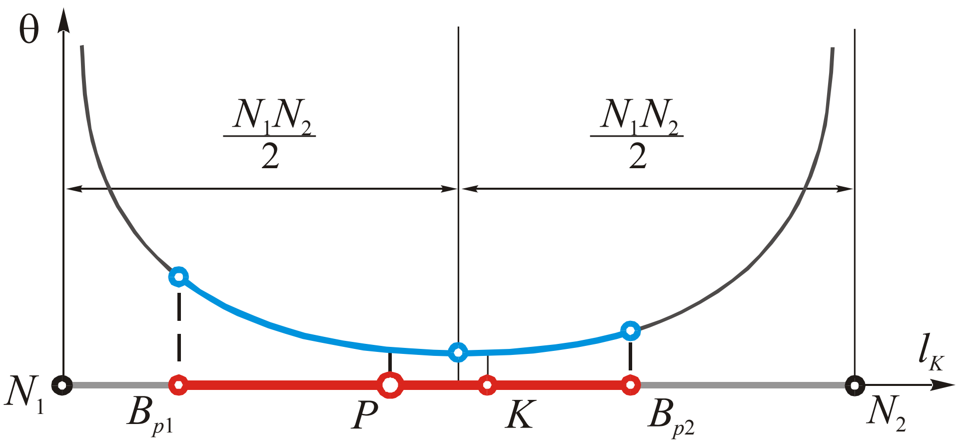


Fig. 9.25. The graph of a change of specific pressure ratio depending on the position of contact point K on the line of action

Specific pressure ratio ϑ depends on shift factors of gears x_1 and x_2.

The more the shift factors are, the less the specific pressure ratio we get. At a designing of an involute gearing we should aim at the ratio ϑ being the less.

9.2.9. Tooth interference. Undercutting and pointing of teeth

Interference is interpenetration of teeth at their meshing out of a line of action.

This effect is accompanied by plastic strains of tooth tip edges and their roots.

The cause of interpenetration is gear manufacturing errors, and it results in their inadmissible approach after assembling. The other cause may be an incorrect choice of a shift factors.

The interference of teeth of the gear blank and the tool in the gear-cutting process results in the undercutting of tooth roots.

Consider the conditions under which tooth undercutting is possible.

Standard pitch diameter of a blank $d = mz$. When the diameter d is unchangeable, the number of teeth, which can be cut, decreases with the increase of module. Tooth dimensions also increase.

Let's define a minimum number of teeth, which can be cut without tool shifting, in order not to have undercutting.

Fig. 9.26 shows rectilinear part of tooth tip of a rack between reference pitch line and addendum line (the distance $h_a^* m$), which forms the involute part of a tooth root of a blank. The addendum line crosses the base tangent on left from the point *N*. That is the engagement of teeth of the rack and the gear blank occurs out of a theoretical line of action, which results in undercutting of tooth roots.

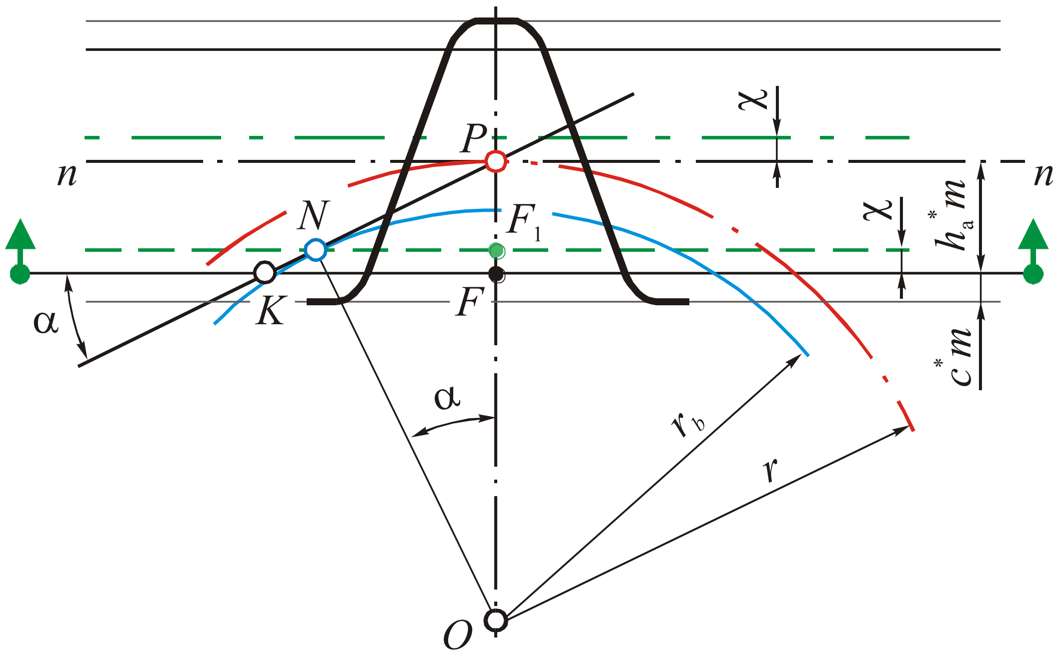


Fig. 9.26. Scheme of rack and blank teeth engagement, when undercutting of roots takes place

Undercutting of teeth at shiftless gear cutting does not take place when:

$$NP \geq KP. \tag{9.20}$$

Taking into account that

$$NP = OP \sin\alpha = \frac{mz}{2} \sin\alpha;$$

$$KP = \frac{PF}{\sin\alpha} = \frac{h_a^* m}{\sin\alpha},$$

condition (9.20) looks like:

$$\frac{mz_{\min}}{2}\sin\alpha \geq \frac{h_a^* m}{\sin\alpha}.$$

Hence

$$z_{\min} \geq \frac{2h_a^*}{\sin^2\alpha}. \qquad (9.21)$$

When $h_a^* = 1$ and $\alpha = 20°$

$$z_{\min} \approx 17.$$

The minimum number of teeth that can be cut without tool shifting so that there is no undercutting $z_{\min} \approx 17$**.**

To cut a gear with teeth number $z < 17$ without their undercutting, we should provide positive tool shift χ, so that addendum line of a rack passes through the point on a base circle of a gear blank, that is on right from the point *N*, or, at least, passes through it (See Fig. 9.26). For this case we have the condition:

$$NP = KP.$$

Here

$$NP = \frac{mz}{2}\sin\alpha; \qquad KP = \frac{PF_1}{\sin\alpha} = \frac{\left(h_a^* - x\right)m}{\sin\alpha}.$$

Then

$$\frac{mz}{2}\sin\alpha = \frac{\left(h_a^* - x\right)m}{\sin\alpha}.$$

Hence

$$z\sin^2\alpha = 2\left(h_a^* - x\right).$$

On the other hand, according to (9.21)

$$\sin^2\alpha = \frac{2h_a^*}{z_{\min}} = \frac{2}{17}.$$

In such way

$$\frac{2}{17}z = 2(1-x).$$

Finally we get:

$$x = \frac{17-z}{17}.$$

At positive shift, as it was already mentioned, a root thickens, but at the same time, tooth tip becomes pointed, which decreases the strength of tooth edge at its engagement with a root of another gear. So positive shift is limited by tooth point thickness, which is proportionate to a module:

$$s_a = s_a^* m.$$

Here s_a^* is tooth point thickness ratio; its magnitude is set depending on heat treatment mode of teeth. The recommendations for the choice of s_a^* are in Appendix 4.

9.2.10. Choice of shift factors. Limiting contours

When choosing shift factors in the process of designing of gearings three conditions should be fulfilled:

– absence of undercutting teeth ($x_1 \ge x_{1\min}, x_2 \ge x_{2\min}$);

– absence of inadmissible pointing of tooth tips ($x_1 \le x_{1\max}, x_2 \le x_{2\max}$);

– continuity of gear meshing ($\varepsilon_\alpha \ge \varepsilon_{\alpha\min}$).

So there is some admissible range for the choice of shift factors:

$$x_{\min} \le x \le x_{\max}.$$

For responsible load-bearing gear trains, in which higher demands are set to the drive operation smoothness, strength, life, shift factors should be chosen in the mentioned range with the consideration of quality indicators of designed gearing. It is rather complicated task, as we should optimize several parameters simultaneously, considering operating conditions of a gearing (speed, loading condition, lubrication conditions, pinion and weal materials and their heat treatment modes etc.).

This task is solved with the help of so-called *limiting contours*.

Limiting contours is a group of lines in coordinate system x_1 and x_2, which limit the admitted range of values of basic rack shift factors for a gearing with some teeth number of a pinion and a weal z_1 and z_2.

For each gearing we can built its limiting contour. Fig. 9.27 shows a limiting contour for spur gearing with teeth number of a pinion z_1=12 and a weal z_2=15 [2].

Here red lines show the bound of admitted range of values of shift factors x_1 and x_2. On the left and down this range is limited by the absence of undercutting conditions ($x_1 = x_{\min 1}$; $x_2 = x_{\min 2}$). And on the right it is limited by continuity of meshing condition $\varepsilon_\alpha \ge \varepsilon_{\alpha \min}$. As it is seen from the figure, lines, which correspond to the condition of inadmissible pointing of tooth tips ($s_{a1} = 0, s_{a2} = 0$), go out of the bounds of the region. It tells that for the gearing with z_1=12, z_2=15, the limitation by $\varepsilon_\alpha = 1$ comes earlier than by pointing of tooth tips.

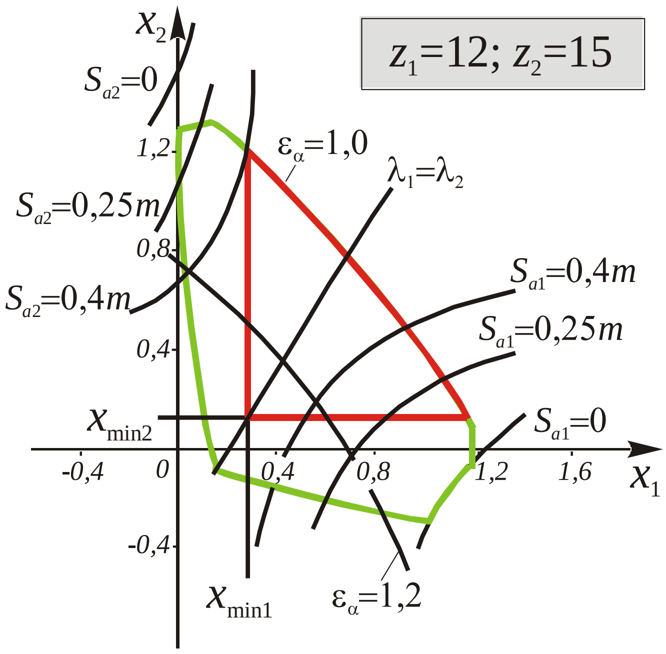


Fig. 9.27. Limiting contours

Green lines show the bounds of dilated admitted range of values of shift factors x_1 and x_2. But such extension is not recommended by the standard.

For optimal choice of shift factors inside the limiting contour are shown isolines, which corresponded to best-case values of quality indicators of gearing for concrete operation conditions.

9.3. SPATIAL GEARINGS

Here we will study geometrical and kinematical peculiarities of main types of spatial gearings commonly used in mechanical engineering: bevel and hyperboloid (specifically worm) gearings. In detail you can study the material by this subject in scientific sources [1, 2, 3, 4, 6 etc.]

9.3.1. Involute bevel gearing

In bevel gearing axoids are cones, axes of rotations of which are intersected (See item 9.2.1). Bevel gearing scheme is shown in Fig. 9.28.

Here OP – is an instantaneous axis, which forms angles δ_{w1} and δ_{w2} with gear axes of rotation. This angles are called *pitch angles*. As pitch cones (axoids) roll over each other without sliding, we may write down:

$$\vec{V}_{P1} = \vec{V}_{P2},$$

or

$$\omega_1 l_{OP} \sin\delta_{w1} = \omega_2 l_{OP} \sin\delta_{w2}.$$

Velocity ratio of a gearing

$$u_{12} = \frac{\omega_1}{\omega_2} = \frac{\sin\delta_{w2}}{\sin\delta_{w1}}. \qquad (9.22)$$

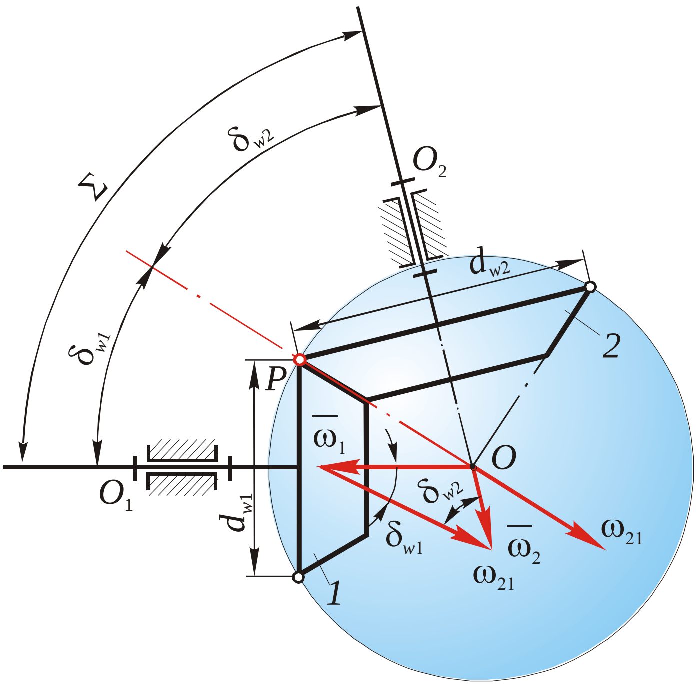


Fig. 9.28. Bevel gearing scheme

Considering that

$$\delta_{w1} + \delta_{w2} = \Sigma, \qquad (9.23)$$

where Σ is a *shaft angle*, let us solve equations (9.22) and (9.23) together:

$$\sin\delta_{w2} = u_{12}\sin\left(\Sigma - \delta_{w2}\right) = u_{12}\left(\sin\Sigma\cos\delta_{w2} - \sin\delta_{w2}\cos\Sigma\right) =$$
$$= u_{12}\sin\delta_{w2}\left(\sin\Sigma\,\mathrm{ctg}\delta_{w2} - \cos\Sigma\right);$$

$$u_{12}\sin\Sigma\,\mathrm{ctg}\delta_{w2} = 1 + u_{12}\cos\Sigma.$$

Hence

$$\mathrm{tg}\delta_{w2} = \frac{u_{12}\sin\Sigma}{1 + u_{12}\cos\Sigma},$$

or

$$\mathrm{tg}\delta_{w1} = \frac{\sin\Sigma}{u_{12} + \cos\Sigma}.$$

Bevel gearing with shaft angle $\Sigma = 90^\circ$ is called a *right-angle bevel gearing*. For such gearing

$$\mathrm{tg}\delta_{w2} = u_{12};$$

$$\mathrm{tg}\delta_{w1} = \frac{1}{u_{12}}.$$

All points of links 1 and 2 (Fig. 9.28) move along spherical paths. The trajectory of the point P is placed on a sphere with a radius OP.

Basis for teeth dimensions measuring is a *pitch cone*. Pitch cone base is a circle, which is in a plane, perpendicular to a bevel axis, and passes through the point P.

Diameter of base of pitch cone or *pitch diameter* is defined by module:

$$d_1 = mz_1;\; d_2 = mz_2.$$

The length of pitch cone generating line OP is called *cone distance* R.

Mating teeth profiles of bevels are generated as involute. As well as in cylindrical gearing, involute profiles are the most manufacturable.

For bevel cutting a generating process as a result of interaction between gear blank and *imaginary generating gear* is used. Flank tooth surfaces of *generating gear* are formed by cutting motion of gear-cutting tool. The most spread is an instrument – rack analogue – with straight cutting edge. Under linear main motion a straight edge forms *flat generating gear* with vertex angle $\delta_{woc} = 90^\circ$ (Fig. 9.29, *a*), or *flat-topped generating gear* with $\delta_{woc} = 90^\circ - \theta_{fwo1}$ (Fig. 9.29, *b*), where θ_{fwo1} is a root angle of a gear, which is cut in gear-cutting process (for details see [1-4, 6]).

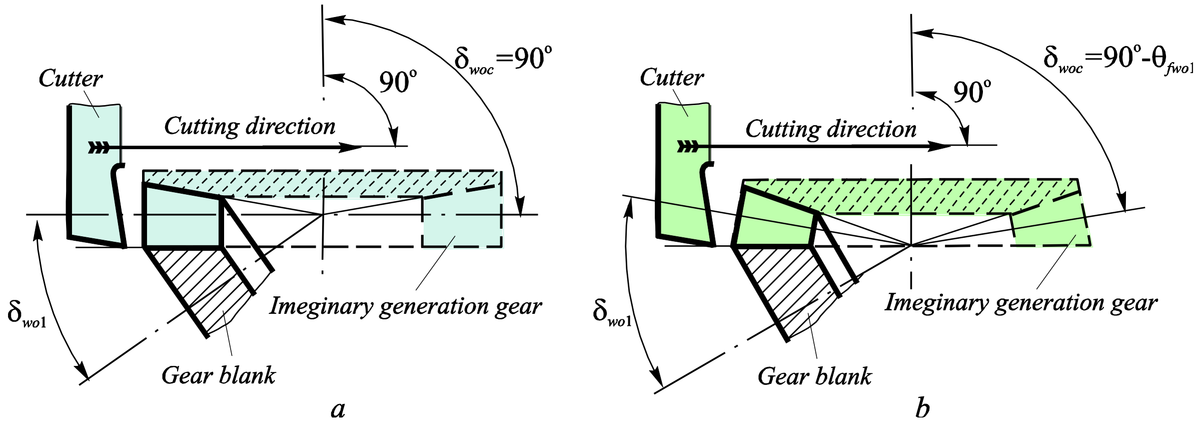


Fig. 9.29. Generating process for bevel cutting: *a* – with flat generating gear; *b* – with flat-topped generating gear

Bevel tooth thickness along generating line is a variable magnitude. Correspondingly, the pitch is variable too (Fig. 9.30): $p_e > p_m > p_i$. Thus, the module along generating line is also variable.

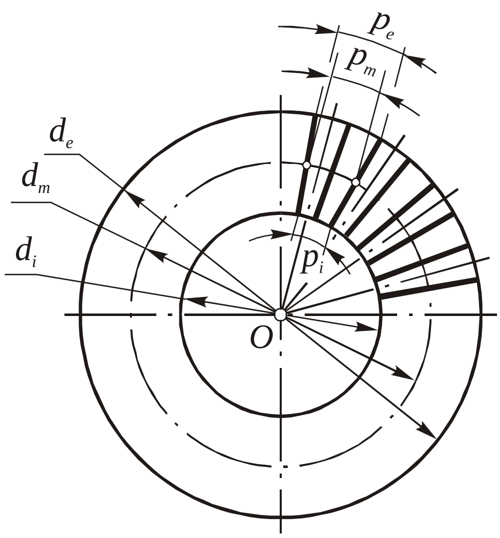


Fig. 9.30. Bevel dimensions

There are *outer module*

$$m_e = \frac{p_e}{\pi},$$

middle module

$$m_m = \frac{p_m}{\pi},$$

and *inner module*

$$m_i = \frac{p_i}{\pi}.$$

The outer module is standardized. This makes size control a lot easier.

The main dimensions of bevel gears should include (Fig. 9.31):

– *outer diameters*: d_e – pitch; d_{fe} – root; d_{ae} – outside;

– *middle diameters*: d_m, d_{fm}, d_{am};

– *inner diameters*: d_i, d_{fi}, d_{ai}.

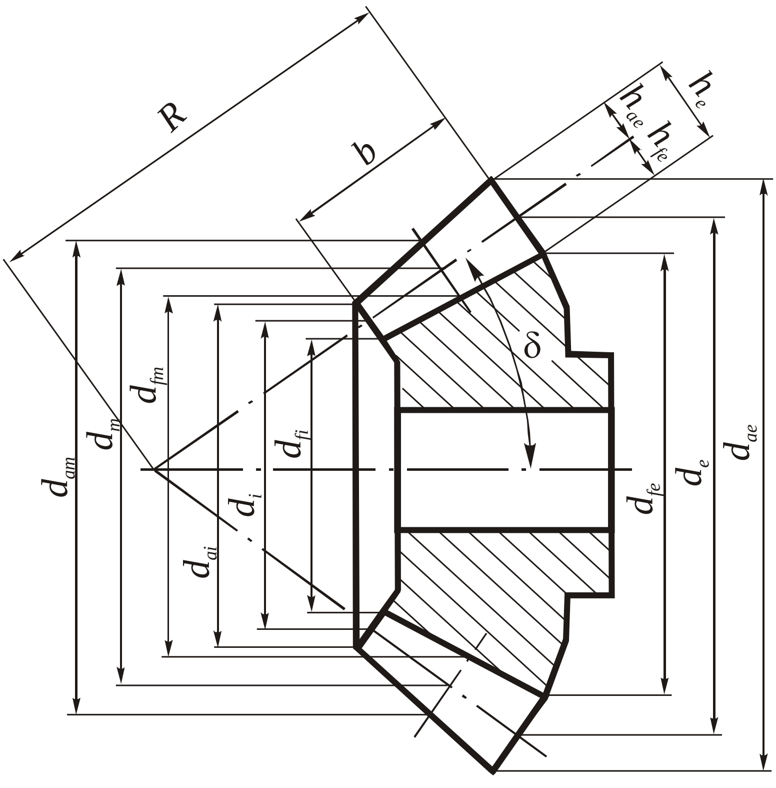


Fig. 9.31. Basic bevel dimensions

For defining of outer diameters of shiftless bevels, we should use following design formulas:

$$d_e = m_e z;$$

$$d_{fe} = m_e z - 2h_{fe} \cos\delta;$$

$$d_{ae} = m_e z - 2h_{ae} \cos\delta.$$

Outer cone distance:

$$R_e = 0,5 \frac{d_e}{\sin\delta}.$$

Here δ – is a pitch angle of set bevel.

Outer tooth depth:

$$h_e = h_{ea} + h_{ef},$$

where $h_{ea} = h_a^* m_e$ is an addendum; $h_{ef} = \left(h_a^* + c^*\right) m_e$ is a dedendum. Here $h_a^* = 1$; $c^* = 0,2$ for shiftless bevels.

A middle module is connected with an outer module by the ratio:

$$m_m = m_e - \left(\frac{b}{z}\right) \sin\delta.$$

Here b is a ring gear width (See Fig. 9.31).

9.3.2. Skew axes gearings

Unlike cylindrical and bevel gearing, here gears under relative motion turn by some axis and slide along it. Such axis is called an *instantaneous screw axis*.

The loci of instantaneous screw axes for each of gears are *screw axoids of relative motion*.

If velocity ratio is constant, so the instantaneous screw axis positions do not change, and screw axoids of relative motion are unparted hyperboloid of rotation (Fig. 9.32). So such gearings are called *hyperboloid gearings*.

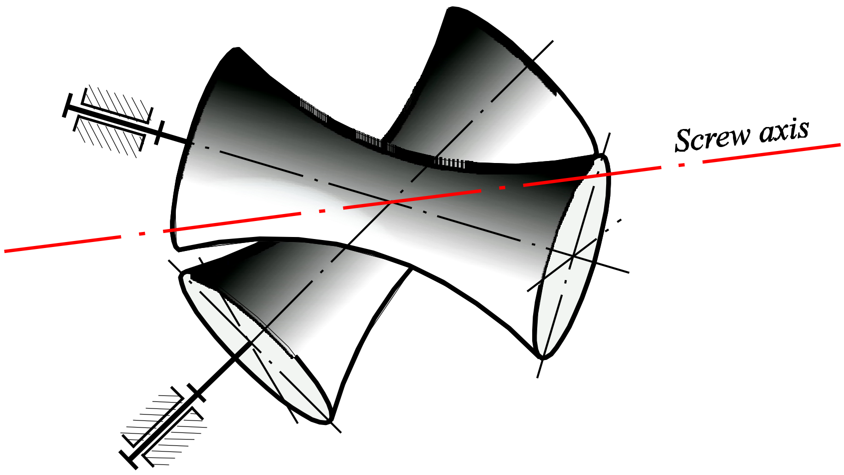


Fig. 9.32. Screw axoids

Unlike cylindrical or bevel gearing, gear pitch surfaces don't coincide with axoids in a shape, but are formed by contact point area of mating tooth profiles. Comparative estimation of efficiency, dimensions and other hyperboloid gearing characteristics showed that sometimes teeth are to be placed in areas distant from screw axoids.

There are first and second types of hyperboloid gearings.

In a case of the first type hyperboloid gearings both mating tooth surfaces may be shaped by the same generating surface, and for the second type hyperboloid gearings generating surface coincides with one of the mating surfaces.

Hypoid and screw gearing. These trains belong to the first type hyperboloid gearings.

Hyperboloid gearing with cone pitch surfaces is called hypoid gearing (Fig. 9.33, *a*), and with cylindrical pitch surfaces – screw gearing (Fig. 9.33, *b*).

Fig. 9.33, *a* and *b* show examples of hyperboloid gearings with axes intersected by 90°, though this angle may not be right.

We should also note that in screw gearing the contact between teeth is occurred in a point.

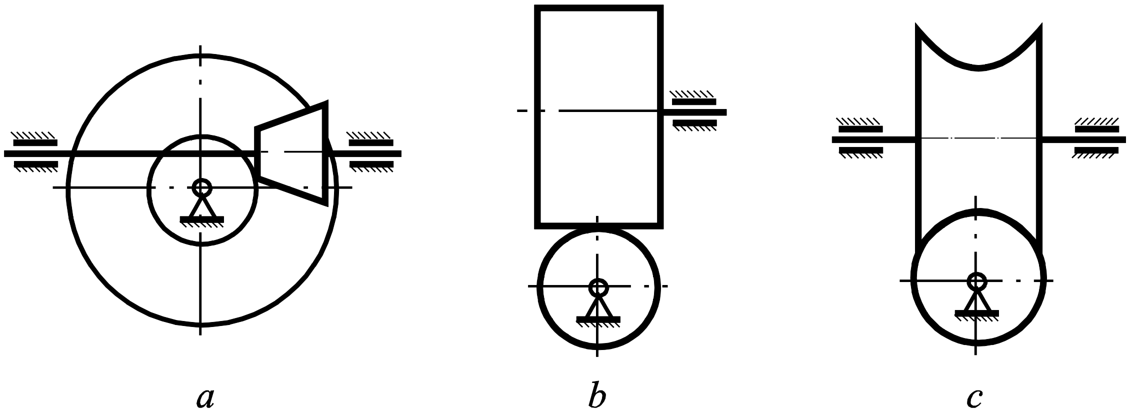


Fig. 9.33. Hyperboloid gearings: *a* – hypoid gearing; *b* – screw gearing; *c* – worm gearing

Worm gearing. Fig. 9.33, *c* shows a worm gearing. This gearing belongs to the second type hyperboloid gearings.

In worm gearings there is a gear teeth linear contact. Here the small gear is called *a worm* (a gear in the form of a screw), which is also called *a worm screw*. The large gear is called *a worm wheel* or *a worm gear*, which is similar in appearance to a helical gear.

The angle between skew axes of gears is always equal to 90º.

As common helical screws, worms can be single-threaded (z_1=1) and multithreaded (z_1>1). Here z_1 is a number of worm starts.

Worm gearings are distinguished by the worm shape: with cylindrical or *single-enveloping worm* (Fig. 9.34, *a*) and *double-enveloping worm* (Fig. 9.34, *b*).

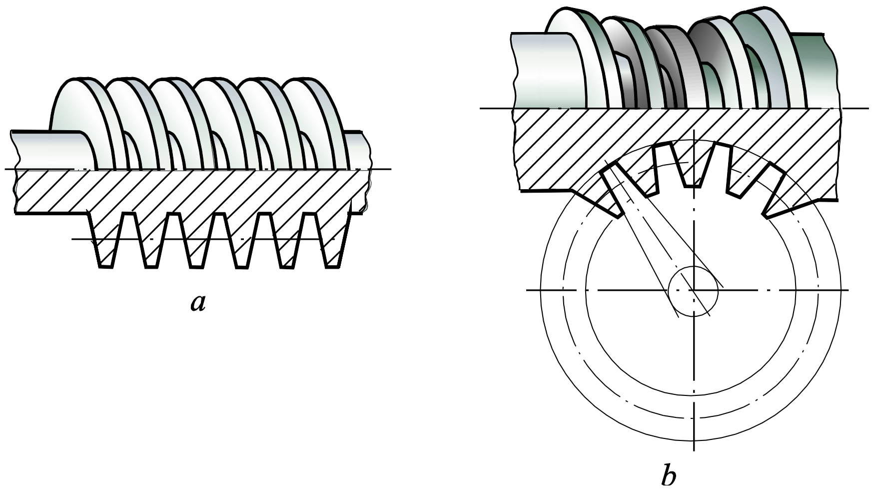


Fig. 9.34. Worm shapes: *a* – single-enveloping worm; *b* – double-enveloping worm

According to a shape of screw tooth surfaces of worms they can be divided into two types: with ruled surface and non-ruled surface. The most spread are two types of worms with screw ruled surface: *archimedean worm* and *involute worm*.

It is easy to cut an archimedean worm, but it is difficult to finish it. Involute worms are more practically feasible.

Fig. 9.35 shows a section of a worm gear pair, which is also called a worm-and-worm pair.

Here d_1 and d_2 are pitch diameters of a worm and a worm wheel.

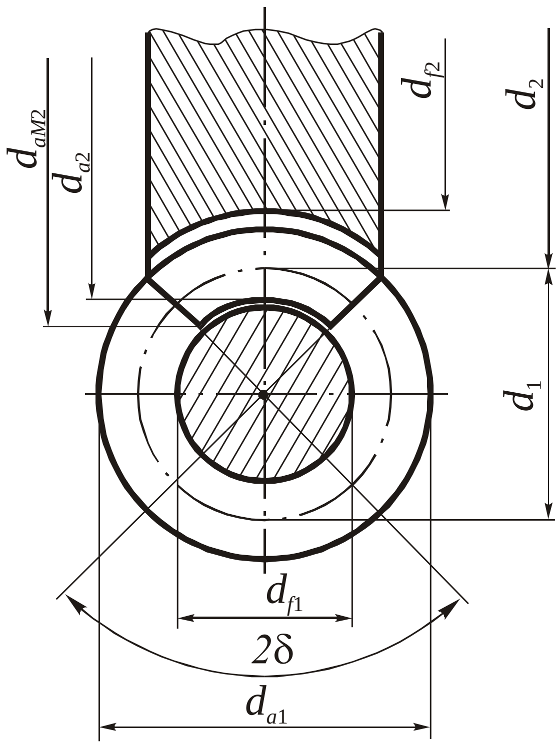


Fig. 9.35. Worm-and-worm pair

Let us study a worm thread sweeping (Fig. 9.36).

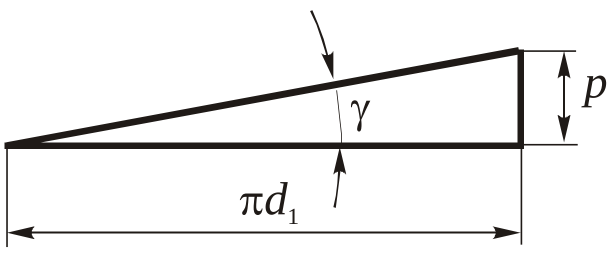


Fig. 9.36. Worm thread sweeping

Lead angle γ is connected with a thread pitch p and a pitch diameter of a worm by formulas:

– for a single thread worm

$$\mathrm{tg}\gamma = \frac{p}{\pi d_1};$$

– for a multithread worm

$$\mathrm{tg}\gamma = \frac{p z_1}{\pi d_1}, \tag{9.24}$$

where z_1 is a number of threads in a worm.

From (9.24) we get:

$$d_1 = \frac{p}{\pi}\frac{z_1}{\mathrm{tg}\gamma} = mq.$$

Here q is a *worm-diameter factor*, which is defined according to DSTU 2458-94 (See Appendixes 5 and 6):

$$q = 6{,}3 \ldots 25.$$

Basic dimensions of worm gearing and its elements are defined by the following formulas (zero shift of a tool):

For a worm:

– pitch diameter of a worm

$$d_1 = mq;$$

– root diameter

$$d_{f_1} = d_1 - 2\left(h_a^* + c^*\right)m;$$

– outside diameter

$$d_{a_1} = d_1 + 2h_a^* m.$$

For a worm wheel:

– pitch diameter of a worm wheel

$$d_2 = mz_2;$$

– root diameter

$$d_{f_2} = d_2 - 2\left(h_a^* + c^*\right)m;$$

– addendum circle diameter

$$d_{a_2} = d_2 + 2h_a^* m;$$

– maximum worm wheel diameter

$$d_{aM_2} = d_2 + d_1\left(1 - \cos\delta\right).$$

Here $\delta = 100\ldots110°$ (See Fig. 9.35).

In previous formulas parameters $h_a^* = 1$; $c^* = 0{,}2$.

Center distance of a worm gearing

$$a_w = 0{,}5\left(q + z_2\right)m.$$

The main kinematic parameter of a worm gearing is a transmission ratio, which is defined by the formula:

$$u_{12} = \frac{z_2}{z_1}.$$

When moving, worm threads slide by worm wheel teeth. It is shown in Fig. 9.37.

Linear velocities of a worm and worm wheel, turned in perpendicular planes, are represented by vectors $\vec{V}_1$ and $\vec{V}_2$ in the velocity diagram (Fig. 9.37). Here $\vec{V}_{slide}$ is sliding velocity. These velocities are connected by vectorial relationship

$$\vec{V}_{slide} = \vec{V}_1 - \vec{V}_2.$$

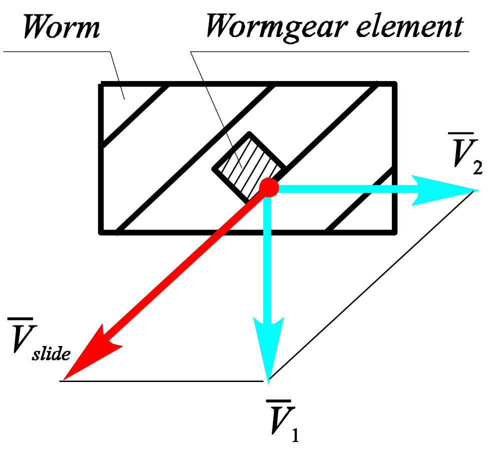


Fig. 9.37. Velocities of pairing elements in worm gearing

We see very hard worm gearing usage: a direction of contact patch displacement do not coincide with sliding velocity direction $\vec{V}_{slide}$. It creates adverse conditions for the formation of fluid wedge. So worm gearing elements are made of antifriction materials. In power trains worms are made of steel, and worm wheels – of iron, bronze, brass, depending on the charge and loading of a train.

Worm gearing coefficient of efficiency is defined in the same way as for screw thread (See Chapter 11):

$$\eta = \frac{\mathrm{tg}\gamma}{\mathrm{tg}(\gamma + \varphi)}.$$

Here γ – is a worm angle; φ – is an angle of friction for set antifriction pair.

More information on hyperboloid gearings can be found in [1-4, 6].

QUESTIONS FOR SELF-TESTING

1. What kind of mechanical gear do we call gear train?
2. What are the main features for gear train classification?
3. What gear trains are called involute gearings? Give examples of noninvolute gearings.
4. What are the main advantages and disadvantages of using involutes as mating tooth profiles for gear wheels?
5. What do we call the base circle of gear?
6. What are the names of centroids in gearing?
7. What line is called the theoretical line of action? What is called an effective line of action?
8. What do we call the pressure angle?
9. What do we call the circular pitch?
10. What is the module of spur gear?
11. What is the standard pitch circle?
12. Do gears have to have the same module to engage?
13. What are the main methods of gear manufacturing?
14. What is the form-cutting method of gear manufacturing?
15. What is the essence of the manufacturing method of cylindrical gears by generating cutting processes?
16. What are the main parameters of the basic rack tooth profile?
17. What relative motions do the tool and the blank carry out in the gear cutting process?
18. What surface do we call the generating one when cutting gear teeth?
19. What gears can be obtained after gear-cutting process depending on the relative position of the basic rack and the blank?
20. The module of gears m=10 *mm*, the number of teeth $z_1 = 10$, $z_2 = 15$. With what minimal shift should the gear be cut to avoid undercutting the profiles of its teeth?
21. What are the main *gearing quality indicators*?
22. How does the shift of the tool in the manufacture of the gear wheels affect the contact ratio?
23. How does the specific pressure ratio change as the tool shift increases when cutting gears? If so, which way? Justify the answer.

24. What should be the specific sliding ratios on roots of gear teeth to maximize the gear train life?
25. What surfaces are called pitch ones in bevel gears? What is the shape of these surfaces?
26. What the bevel gear train is called orthogonal?
27. Which module in bevel gear is standardized?
28. What are hyperboloid gears?
29. Do pitch surfaces in hyperboloid gears coincide with axoids?
30. To which gearings do worm gearings belong, taking into account the form of axoids?

Chapter 10. CAM MECHANISMS

In order to provide an operating mode of many machines we should often add to their structure mechanisms, the motion of output links of which is carried out by strictly set laws, coordinated with motion laws of other mechanisms. In previous chapters we studied different linkages and gear trains and it is evident that the range of functions that can be fulfilled by such mechanisms are somewhat limited. This problem is very easily solved with the help of compact *cam mechanisms*.

Cam mechanisms belong to three-link mechanisms, which consists of a fixed frame and two movable links, which form between themselves a higher kinematic pair, and with the fixed frame – two lower kinematic pairs.

Input link in cam mechanism is, as a rule, represented by *cam*, so to say a link with a higher pairing element, made in a form of surface of variable curvature.

Output link – *follower* (because it is caused to "follow" the surface of the cam) – carries out back-and-forth motion, swinging motion or space motion according to cam mechanism construction. Translating follower is called a *pusher*, swinging follower – *is a rocker*, *rocker follower or oscillating follower*.

10.1. TYPES OF CAM MECHANISMS

There are planar and space cam mechanisms. We will pay major attention to planar cam mechanisms, as the most prevailing in technique.

10.1.1. Planar cam mechanisms

As for any other planar mechanisms, in a planar cam mechanism the whole of its points move in parallel planes.

Planar cam mechanisms are classified according to their kinematics and design:

- **According to nature of motions, realized by a cam and a follower:**

– Reciprocal sliding of a cam transforms into reciprocal sliding of a follower (Fig. 10.1, *a*);

– Reciprocal sliding of a cam transforms into swinging motion of a follower (Fig. 10.1, *b*);

Cams are shown in Fig. 10.1, *a* and *b* are often called wedge cams or sliding cams.

– Rotary motion of a cam transforms into reciprocal sliding of a follower. This type of cam mechanisms are made with followers on line of cam's axis (Fig. 10.1, *c*) and with offset follower or with the eccentricity (Fig. 10.1, *d*);

– Rotary motion of a cam transforms into swinging motion of a follower (Fig. 10.1, *e*).

These types of cam mechanisms are often referred to as plate cams.

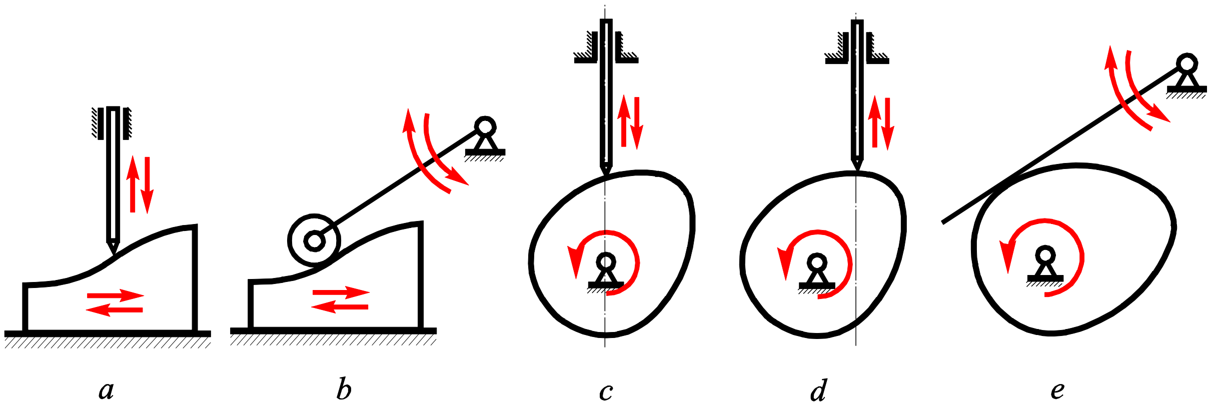


Fig. 10.1. Types of cam mechanisms according to their kinematic: sliding cams (*a*, *b*); rotating cams (*c*, *d*, *e*)

- **According to the type of a follower:**

Some of the types of follower configurations most commonly used with cams are shown in Fig. 10.1 and Fig. 10.2. The followers in Fig. 10.1, *a*, *c*, *d* and Fig. 10.2 are sliding or translating followers. Fig. 10.1, *b* and *e* show swinging followers.

Moreover we also consider cam mechanisms:

– with pointed follower (Fig. 10.1, *a*, *c*, *d*);

– with roller follower (Fig. 10.1, *b*; Fig. 10.2 *a*);

– with spherical mushroom follower (Fig. 10.2, *b*);

– with flat-faced follower (Fig. 10.1, *e*; Fig. 10.2, *c*).

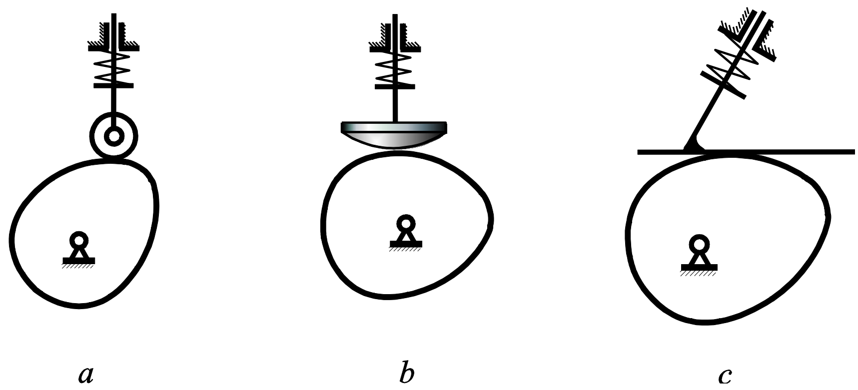


Fig. 10.2. Types of cam mechanisms according to the type of a follower: *a* – with roller follower; *b* – with spherical mushroom follower; *c* – with flat-faced follower

10.1.2. Space cam mechanisms

There are a great number of schemes of space cam mechanisms. The most frequent are:

– with barrel cam (Fig. 10.3, *a*);

– with conical cam (Fig. 10.3, *b*);

– with hyperboloid cam (Fig. 10.3, *c*);

– with conoid cam (Fig. 10.3, *d*).

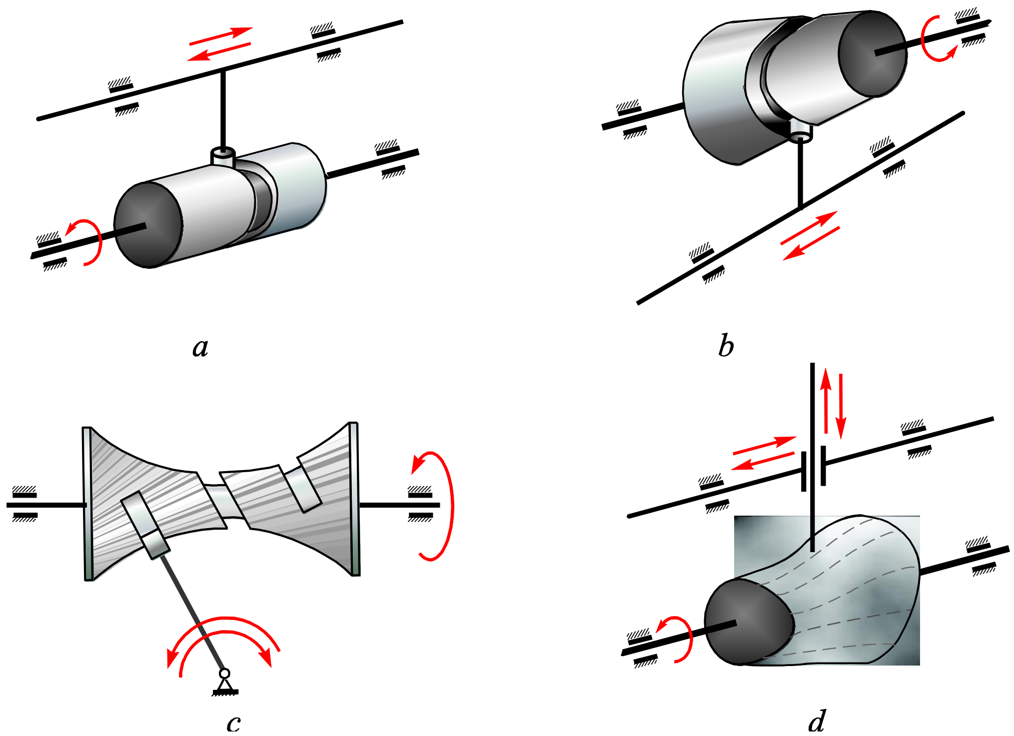


Fig. 10.3. Space cams: *a* – with barrel cam; *b* – with conical cam; *c* – with hyperboloid cam; *d* – with conoid cam

10.2. MAINTAINING CONTACT BETWEEN CAM AND FOLLOWER

For maintaining contact between cam and follower we need to assure "cam-follower closing". Theoretically a closing in higher kinematic pair may be realized under follower dead weight. But this method of a closing is unreliable taking into account the inertia imposed on a follower, which can separate a follower from a cam, and that is inadmissible.

In cam mechanisms positive closing is used: force closure and form closure.

10.2.1. Force closure

Besides, the usage of dead weight of a follower should also be considered as force closing methods for higher pairs in a cam mechanism. In case of a force closure springs, liquid or gas pressure are used. Fig. 10.2 and Fig. 10.4, *a* show schemes of spring usage for force closing of higher kinematic pairs in different types of cams.

As it was mentioned, the follower is influenced by inertia, which can separate a follower from a cam, and so break an assigned motion law. In order to prevent the unlocking of a higher kinematic pair, load-deformation curve of a spring should be chosen as that maximum possible inertia acting on a follower was not more than a minimum elastic force in a spring (Fig. 10.4, *b*):

$$F_{el_{\min}} > F_{a_{\max}} .$$

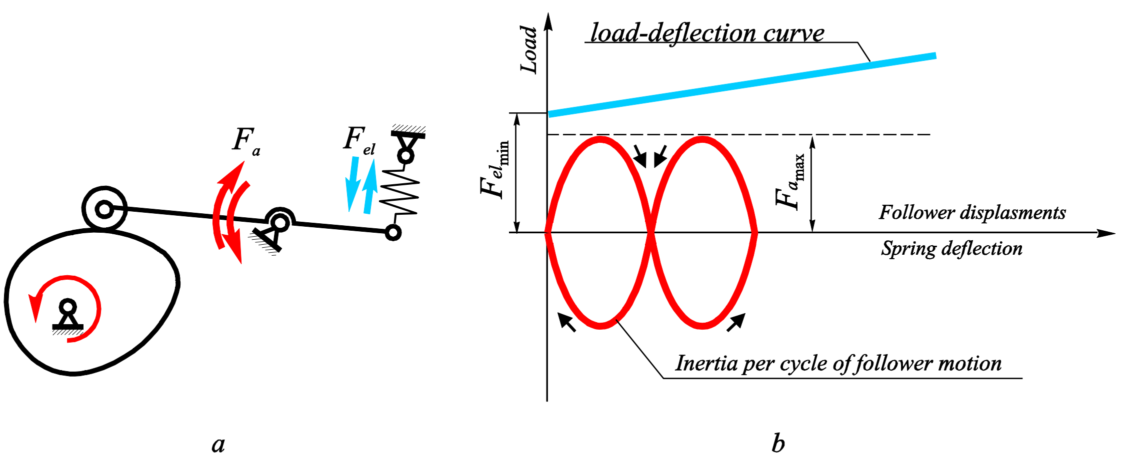


Fig. 10.4. Force closure for cams: *a* – scheme of a cam; *b* – force curves

10.2.2. Form closure

At the form closure a possibility to separate one link from another is eliminated by an insertion of a redundant constraint into the scheme of a mechanism. This constraint must be passive, i.e. doesn't change the motion freedom of a mechanism.

One of the most spread ways of form closure is the usage of a groove or track that is machined in the face of the disk so that as the disk rotates, the follower is forced up or down in a prescribed manner (Fig. 10.5, *a*). This is referred to as a *face cam* or a *track cam.*

Complexion of an accurate production of a groove and impact of a roller with a groove contributed to the appearance of a *conjugate cam* (Fig. 10.5, *b*). In these mechanisms the output link consist of two rigidly bound followers interacts with two rigidly bound plate cams. Their shapes, although different from each other, must be carefully coordinated with each other.

Instead of conjugate cams they use *diametrical cam* (Fig. 10.5, *c*), in which an arbitrary profile can be made only for a part of cam lobe. Another part of a cam lobe is defined from the condition of provision of cam touching to another plane (for the formation of a redundant constraint). The easiest and the most spread among such type of mechanisms is a mechanism, in which diametrical cam is an eccentric (Fig. 10.5, *c*).

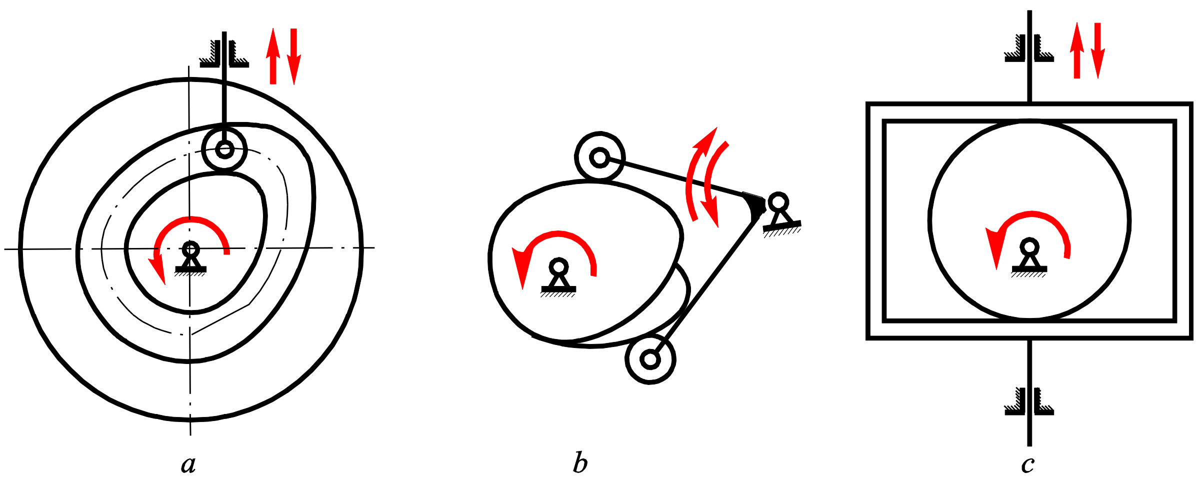


Fig. 10.5. Form closure in a cam mechanism: *a* – face cam;
b – conjugate cam; *c* – diametrical cam

10.3. CENTRAL ADVANTAGES AND DISADVANTAGES OF CAM MECHANISMS

The advantages of cam mechanisms:

– exact reproduction of a given displacement, velocity and acceleration laws of an output link (machines can run constantly on repetitive tasks);

– compactness (for a given function, a cam mechanism almost always occupies less space than does a linkage system);

– simplicity of designing and fast and accurate manufacturing.

The disadvantages of cam mechanisms:

– in a higher kinematic pair there is a high specific pressure, and as a result, higher wear of pairing elements, so there is a need of strengthening treatment, more careful attention must be pained to surface finish and lubrication of link acting faces;

– in the course of a mechanism work, in a case of an out-of-tolerance manufacturing of its elements, there is a loud noise, especially when a follower reverses its movement;

– the need of a closing of a higher kinematic pair makes the mechanism constructively more complicated.

10.4. CAM GEOMETRY

Let’s study cam geometry on the example of admission valve motion control of reciprocating motor (Fig. 10.6). During one turn of a cam we have four phases of valve work.

Phase 1 – valve 3 opening for working medium admission into engine combustion chamber. The cam 1 turns by the angle $\varphi_{вд}$, cam-follower contact point 2 moves along an arc a_0a_1.

Phase 2 – valve is fully opened and is in stationary and outermost position from the pivot point of a cam. The contact point moves in the line of a cam contour by the circular arc a_1a_2 with radius $R_{\max}$, rested upon the central angle $\varphi_{\textit{д}}$ (so-called *high dwell angle*).

Phase 3 – valve closing. Under a spring 4 action valve 3 returns from the outermost position to the initial one, cam-follower contact point moves along the arc a_2a_3, and cam turns by the angle $\varphi_{\textit{пв}}$.

Phase 4 – valve is closed and is in the nearest position from the pivot point of a cam. The contact point slides by the circle arc a_3a_0 with radius $R_{\min}$, and the follower stops. The cam turns by the angle $\varphi_{\textit{б}}$. In the course of a further cam turning, when the contact point reaches the point a_0, the cycle repeats.

The enumerated turning angles are called phase angles of a cam: $\varphi_{\textit{вд}}$ – a rise angle; $\varphi_{\textit{д}}$ – a high dwell angle; $\varphi_{\textit{пв}}$ – a return or fall angle; $\varphi_{\textit{б}}$ – a low dwell angle.

A total cam angle of rotation per cycle is equal to 2π. Hence:

$$\varphi_{\textit{вд}} + \varphi_{\textit{д}} + \varphi_{\textit{пв}} + \varphi_{\textit{б}} = 2\pi.$$

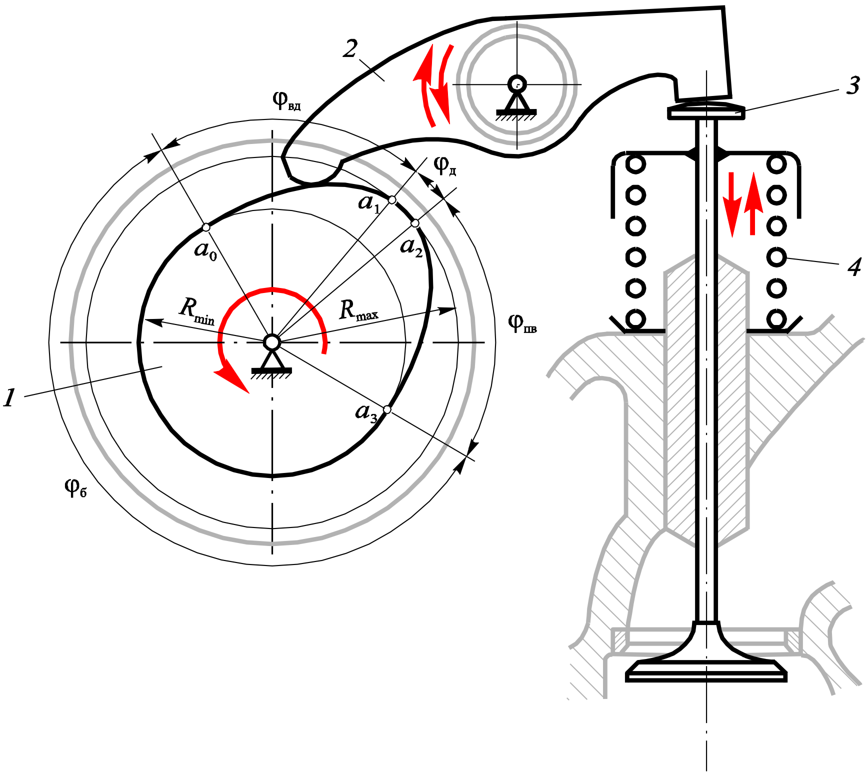


Fig. 10.6. Mechanism of admission valve motion control of reciprocating motor

A working angle:

$$\varphi_p = \varphi_{\text{вд}} + \varphi_{\text{д}} + \varphi_{\text{пв}}.$$

In some cases there can be no dwell angles $\varphi_{\text{д}}$ and $\varphi_{\text{б}}$.

10.5. KINEMATIC ANALYSIS OF CAM MECHANISMS

The general problem of kinematic analysis of cam mechanisms is the definition of follower displacements, velocities and accelerations (or their analogs) under a given cam motion law.

As well as for other planar mechanisms, for cam mechanisms we apply analytical and graphical-analytical methods.

Let us study the examples of kinematic analysis of some cam mechanisms by graphical-analytical method.

Example 10.1. To determine the displacement of point *A* of translating in-line pointed follower at the cam angle of rotation φ with angular velocity ω_1 (Fig. 10.7).

We'll use the inversion principle. Let us give to the whole mechanism angular velocity $-\omega_1$. The cam 1 stops, and the follower 2 with the fixed frame turn around the axis O with angular velocity $-\omega_1$ and simultaneously moves along the axis x-x.

Let us turn the frame with the follower in a direction of angular velocity $-\omega_1$ by the angle φ_{01}. The point A of the follower takes the position $\overline{A_1}$. The magnitude of the follower linear displacement can be found when building an arc with a radius $r = O\overline{A}_1$ to the intersection with the axis x-x in the point A_1. Here a segment A_0A_1 is a searched follower displacement.

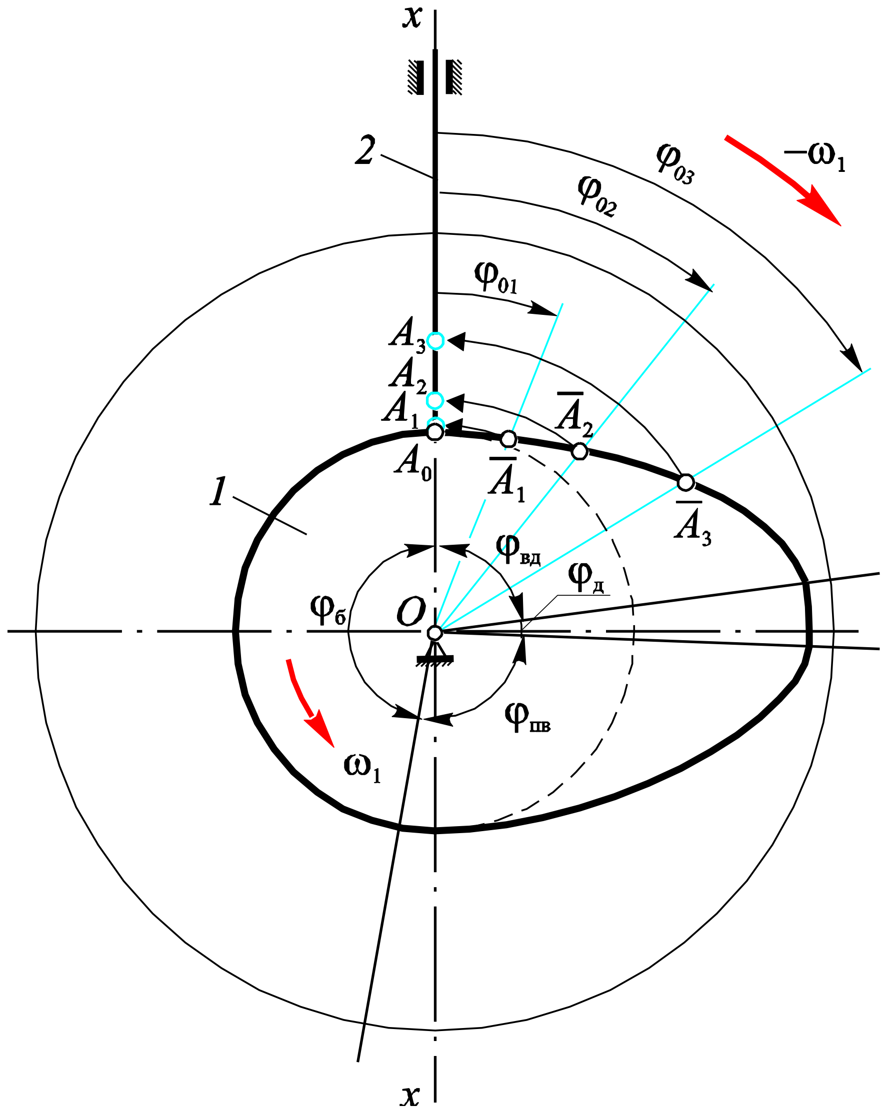


Fig. 10.7. The cam mechanism with translating in-line pointed follower

The obtained displacement is shown in the graph $S_2 = S_2(\varphi)$ *in the form of a segment* $1-1' = \dfrac{\mu_l \cdot A_0A_1}{\mu_s}$ *(Fig. 10.8), where* μ_s *– is a scale factor for an axis of displacement.*

When we turn the ground with the follower around the point O, we get its consistent positions $\bar{A}_2$, $\bar{A}_3$ *etc., which correspond to the angles* $\varphi_{02}, \varphi_{03}$ *etc.*

Showing in scale μ_S *the obtained displacements in the graph* $S_2 = S_2(\varphi_1)$, *we get follower displacement diagram per cycle (Fig. 10.8). It is the graph of follower position function.*

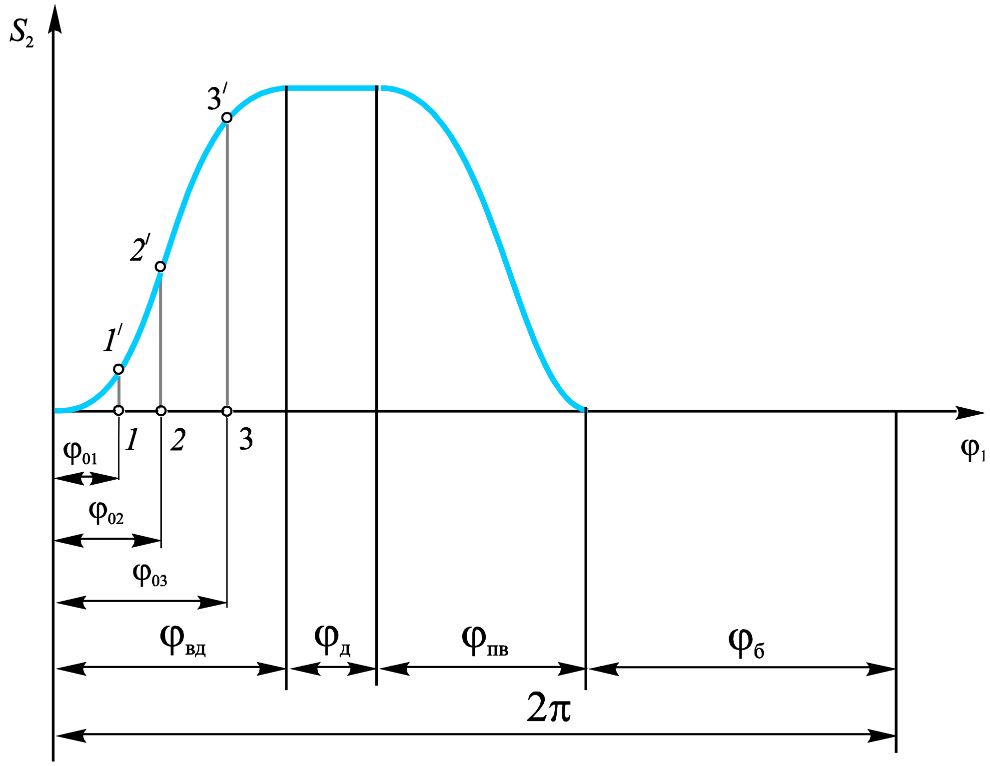


Fig. 10.8. Graph of follower position function

If we have a roller follower, so the analysis is reduced to the previous one, when considering a curve, equidistant to acting profile of a cam – cam surface (Fig. 10.9). This line forms theoretical profile of a cam and is called a pitch curve. All necessary constructions are made for the theoretical profile.

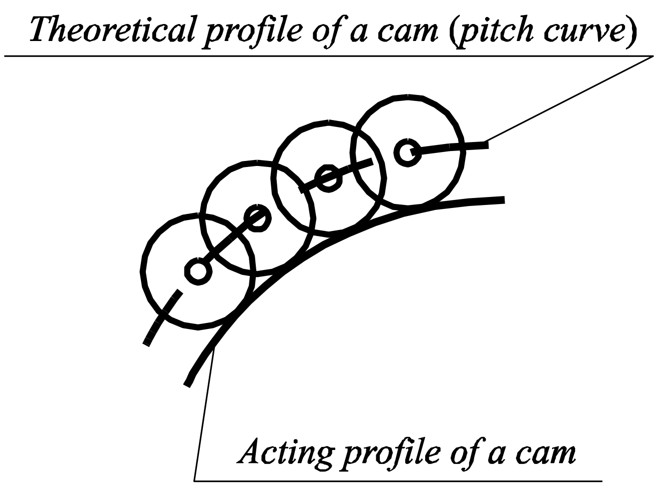


Fig. 10.9. Cam surface formation

<u>Example 10.2.</u> To carry out a kinematic analysis of a cam mechanism with flat-face swinging flower (Fig. 10.10).

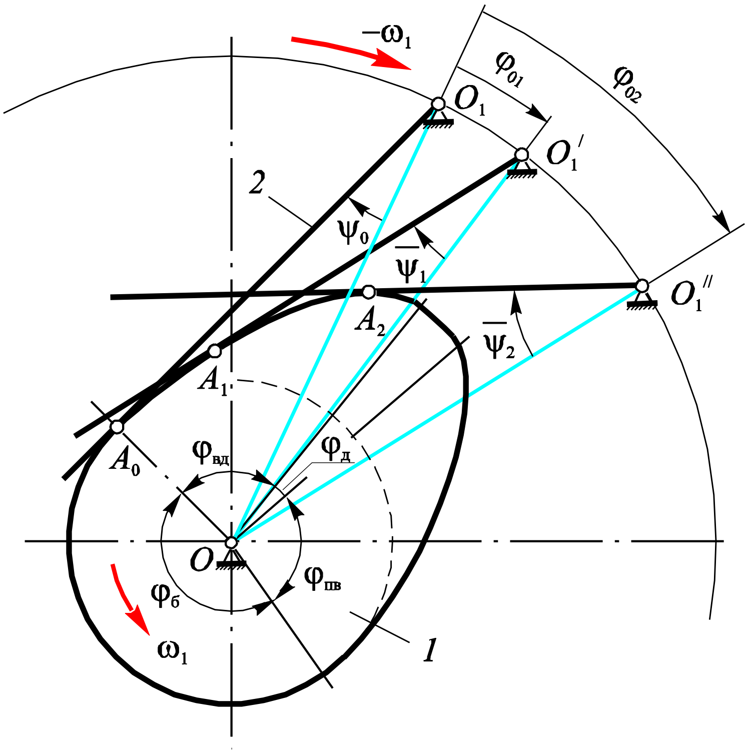


Fig. 10.10. The cam mechanism with flat-face swinging flower

Let's start kinematic analysis of a mechanism with the position corresponded with the beginning of a follower (rocker) rise phase. In this position the rocker 2 touches the cam profile 1 in the point A_0 (See Fig. 10.10). The segment OO_1 is considered to belong to the frame.

For the analysis, as well as in the previous example, we will use the inversion principle. We turn the frame by some angle φ_{01} in direction $-\omega_1$. The segment OO_1 takes position OO_1'. Position of the rocker is defined, when building from the point O_1' a segment that touches the cam profile in the point A_1.

The magnitude of travel of the rocker in this position is defined by angle change of rocker oscillation ψ between segments OO_1 and O_1A_0:

$$\psi_1 = \overline{\psi}_1 - \psi_0.$$

For the second position we have:

$$\psi_2 = \overline{\psi}_2 - \psi_0.$$

Making the same constructions for a set of the mechanism serial positions, and defining in each of them the angle of rocker oscillation, we can construct follower displacement diagram per cam motion cycle in coordinates $\psi - \varphi_1$, which is the same as one shown in the Fig. 10.8 for a cam with a translating follower.

Both in the first and second cases the velocity and acceleration analogue diagrams of a follower should be obtained via graphical differentiation of follower displacement diagrams.

10.6. CAM DESIGN

10.6.1. General remarks

Cam design problem consists in determination of a cam surface by known link dimensions, scheme of a mechanism, and given motion law of driving and driven links.

As a rule driving and driven link motion is given analytically as motion equations or graphically in the form of diagrams. The nature of motion equations or diagrams can be different, depending on given motion conditions. In respect to ease of realization, it is better to set diagrams or motion equations as functions of cam angle of rotation (position function).

Fig. 10.11 shows some examples of diagrams of follower motion (*cam's SVA diagrams*). Displacement diagrams $S_2 = S_2(\varphi_1)$ are shown here in the form of straight lines (Fig. 10.11, *a*), straight lines passed on to circular arcs (Fig. 10.11, *b*), fitted parabolas (Fig. 10.11, *c*) and cosine curve (Fig. 10.11, *d*).

When differentiating these diagrams, we obtain diagrams of follower velocity analogues $\frac{dS_2}{d\varphi_1} = \frac{dS_2}{d\varphi_1}(\varphi_1)$, and when differentiating them once again – diagrams of follower acceleration analogues $\frac{d^2S_2}{d\varphi_1^2} = \frac{d^2S_2}{d\varphi_1^2}(\varphi_1)$.

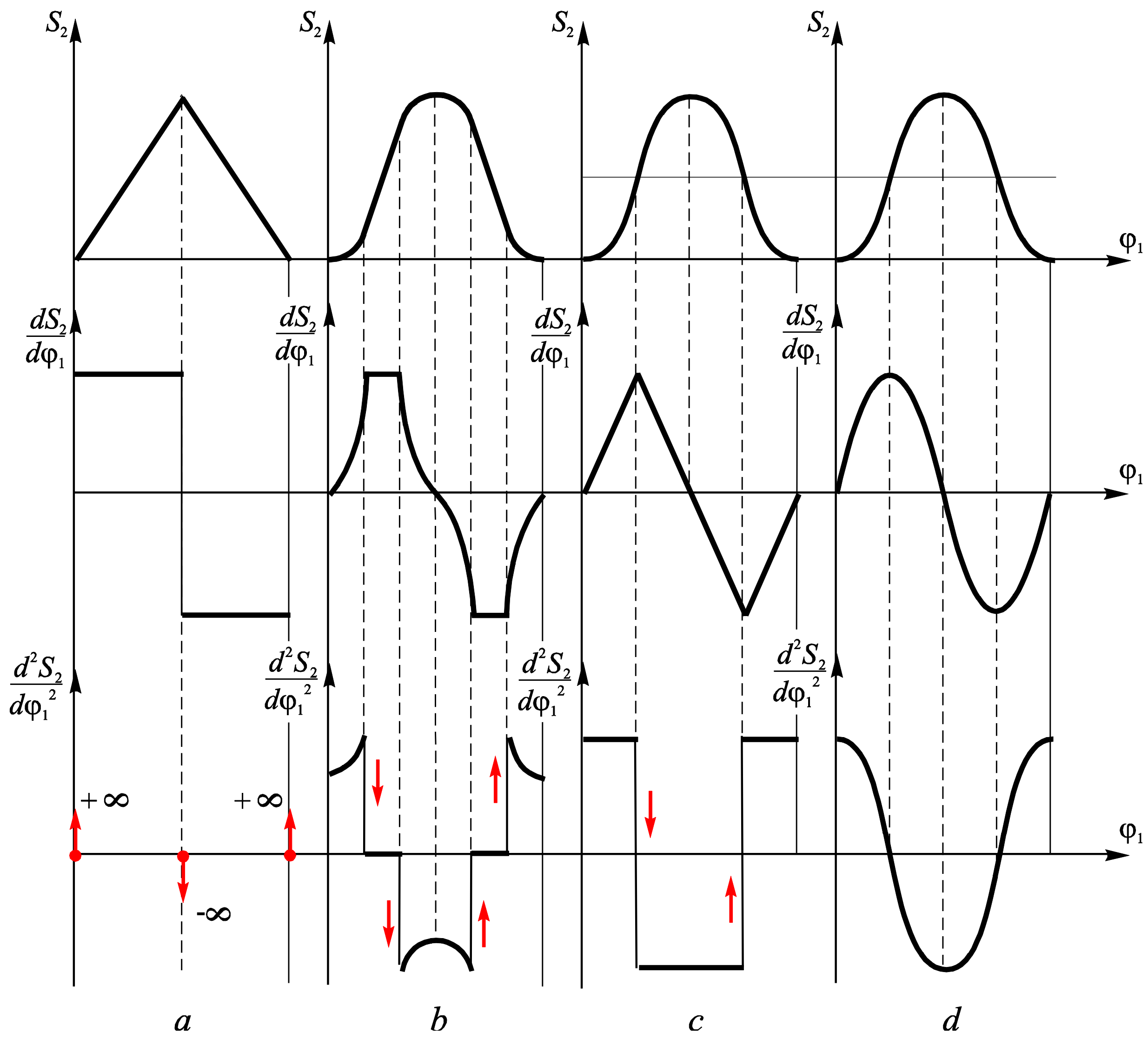


Fig. 10.11. Cam's SVA diagrams: with "hard" impacts presence (*a*); with "soft" impacts presence (*b*, *c*); without impacts (*d*)

Fig. 10.11, *a* shows that under the linear law of follower displacements, in theory the acceleration tends to infinity in points of motion reversal. In this moment there are so-called *"hard" impacts*.

For motion laws, represented in Fig. 10.11, *b* and *c*, under follower motion reversal there is saltatory variation of acceleration. It means that there are also impacts, but pressure change occurs immediately to finite size, rather than to infinity, as under hard impacts. Such impacts are called *"soft" impacts*.

If the law of follower motion is given by a diagram in the form of a cosine wave (Fig. 10.11, *d*), there are no jumps in the velocity and acceleration diagrams, which means that there are no any impacts.

From the point of view of dynamics the last example of laws of follower motion is the most acceptable among all represented ones.

As a rule at cam design in order to avoid impacts it is set the law of follower acceleration analogue variations after a preanalysis of its motion phases. And then by means of integration we'll obtain diagrams of follower velocities and displacements (Fig. 10.12).

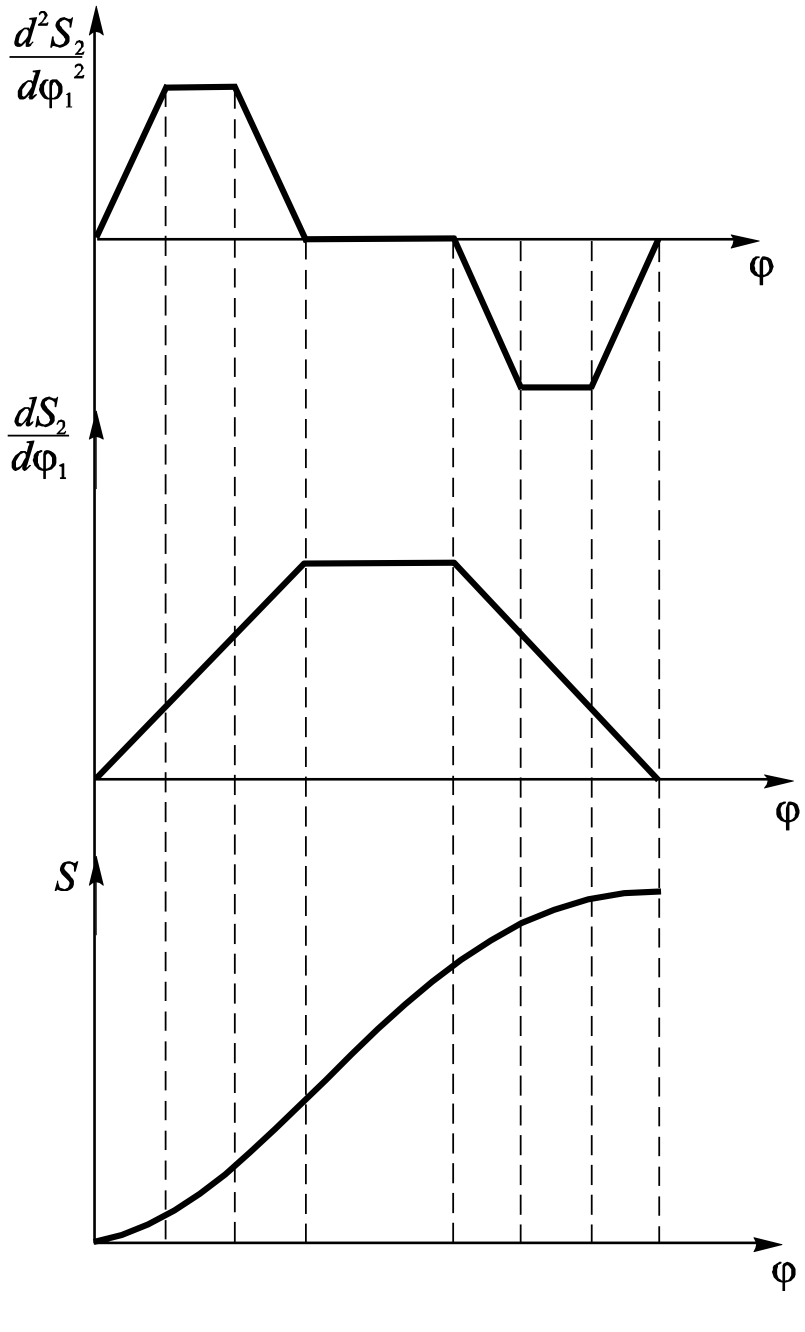


Fig. 10.12. Cam's SVA diagrams

Cam design is carried out in two steps: determination of minimum cam sizes (*dynamic synthesis of a cam*) and determination of cam surface (*kinematic synthesis of a cam*).

10.6.2. Dynamic synthesis of a cam mechanism

Dynamic synthesis problem is to determine minimum cam sizes (minimum radius of a cam, minimum length of a rocker, if it is not set, etc.).

Let us study some peculiarities of dynamic synthesis of some types of cam mechanisms.

10.6.2.1. Cam mechanism with translating roller follower

Let's analyze conditions, which are to be considered when choosing minimum dimensions of such mechanism.

Fig. 10.13 shows a cam with offset follower.

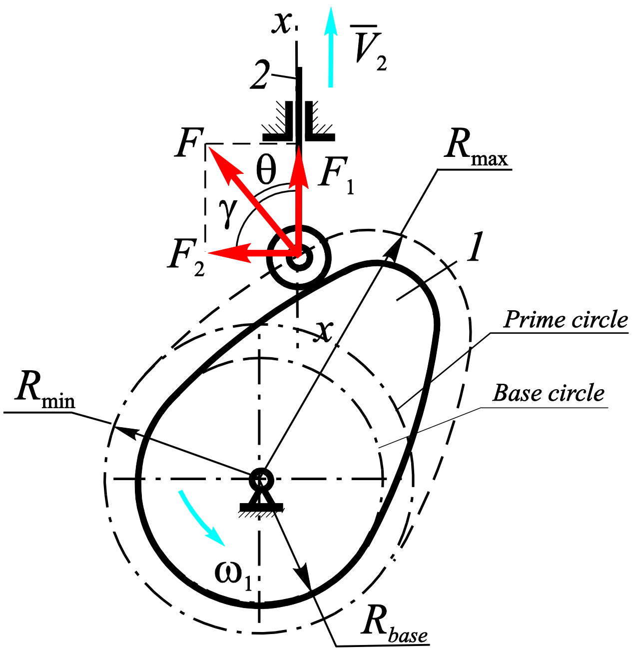


Fig. 10.13. Cam mechanism with offset translating roller follower

As it is known, pressure in a higher kinematic pair is directed along a common normal in a contact point of profiles. Let us decompose the force $\overline{F}$ on two components: along a follower line x – x (force $\overline{F}_1$) and along a perpendicular to the line $x - x$ (force $\overline{F}_2$). Considering the theoretical profile of a cam, here forces are reduced to the contact point of pitch curve of a cam (dotted line) with a follower nib which coincides with the roller center of rotation.

The angle θ in the Fig. 10.13 is the pressure angle.

Pressure angle in a cam mechanism is the angle between a force direction and a velocity direction of a point of this force application.

It is understandable that the more the angle θ is, the more the force F should be, in order to overcome resistance forces, exerted on a follower and make it move.

It means that under some angle θ_{max} seizure of cam follower is possible.

The angle γ, which complements the pressure angle θ till 90°, is called motion transfer angle.

The magnitude of the angle θ is limited, on the one hand, by the condition of the absence of seizure (it means that it cannot be as big as possible), but on the other hand, – by the coefficient of efficiency of a mechanism, which increases with the decrease of the angle θ. The second condition is decisive: usually the optimal angle θ is chosen from the condition of optimal efficiency, but it should be less than one obtained from the absence of seizure.

For translating roller follower:

$$\theta_{max} = 30° \text{ or } \gamma_{min} = 60° .$$

It is understandable that under the set law of motion, in particular follower stroke value, the angle θ will be defined by the rate of rise of a cam profile from R_{min} to R_{max} (See Fig. 10.13). Minimum radius of a cam R_{min}, which is called as prime radius, in its turn determines the dimensions of the cam under the same follower stroke.

Let us study the methodology of determination of prime radius of a cam for this type of cam mechanism.

Fig. 10.14, *a* and *b* shows kinematic scheme and velocity diagram of the mechanism respectively.

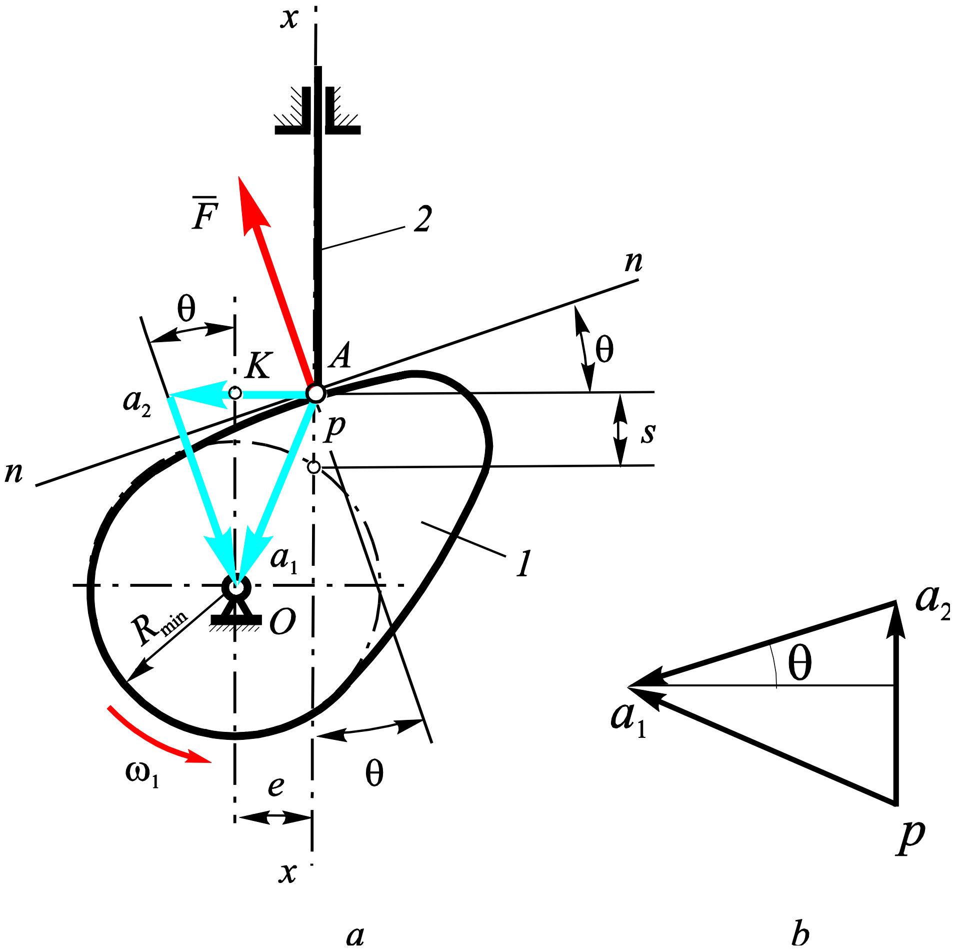


Fig. 10.14. Cam with offset pointed follower: *a* – kinematic scheme; *b* – velocity diagram

Let us combine the pole of the velocity diagram *p* with a point *A* of the mechanism, and a point a_1 – with a point *O*, and on this segment construct the velocity diagram turned by 90°. The scale of this diagram

$$\mu_V = \frac{V_A}{OA} = \frac{\omega l_{OA} \cdot \mu_l}{l_{OA}} = \omega \mu_l. \tag{10.1}$$

Then

$$pa_2 = \frac{V_{A_2}}{\mu_V}. \tag{10.2}$$

Here

$$V_{A_2} = \frac{dS}{dt} = \frac{dS}{d\varphi}\omega. \tag{10.3}$$

We put (10.1) and (10.3) into (10.2):

$$pa_2 = \frac{\frac{dS}{d\varphi}\omega}{\omega\mu_l} = \frac{1}{\mu_l}\frac{dS}{d\varphi}.$$

That is the velocity diagram superposed with the kinematic scheme (Fig. 10.15), can be considered as an analogue velocity diagram constructed in the same scale as the kinematic scheme of the mechanism. Then each segment of an analogue velocity diagram should be summed with any segment of the kinematic scheme.

As a result, when studying the triangle a_1ka_2, we may write down:

$$\mathrm{tg}\theta = \frac{pa_2 - e}{Ok}$$

or

$$\mathrm{tg}\theta = \frac{\frac{dS}{d\varphi} - e}{S + \sqrt{R_{\min}^2 - e^2}}. \tag{10.4}$$

For a follower on line of cam's axis $(e = 0)$:

$$\mathrm{tg}\theta = \frac{\frac{dS}{d\varphi}}{S + R_{\min}}. \tag{10.5}$$

Let us consider the procedure of determination of prime radius of a cam based on geometrical solution of equations (10.4) and (10.5).

In this task laws of follower motion $S - \varphi$ and $\frac{dS}{d\varphi} - \varphi$, its stroke $h_{\max}$ and permissible pressure angle $\theta_{\max}$ or motion transfer angle $\gamma_{\min}$ should be set.

1. We construct the follower motion cyclogram in coordinates $S-\frac{dS}{d\varphi}$, excluding from follower motion diagrams $S-\varphi$ and $\frac{dS}{d\varphi}-\varphi$ the parameter φ (Fig. 10.15). The value of scales μ_S and $\mu_{\frac{dS}{d\varphi}}$ of the cyclogram (Fig. 10.15, *b*) should be equal.

At the force closure we may limit to construction of an cyclogram only for the rise angle $\varphi_{вд}$ (Fig. 10.15), and at the form closure we should construct an cyclogram for the total cam angle of rotation (Fig. 10.16).

2. We determine the prime radius of a cam $R_{\min}$ given permissible pressure angle $\theta_{\max}$. For this we should build a tangent to cyclogram $S-\frac{dS}{d\varphi}$ under the angle $\theta_{\max}$ to the axis *S*. For a follower on line of cam's axis ($e=0$) the point of its intersection with the axis *S* defines the magnitude $R_{\min}$. Really, from the triangle BB_1B_2

$$\mathrm{tg}\theta=\frac{B_1B_2}{BB_2}=\frac{\frac{ds}{d\varphi}}{s+OB}.$$

When comparing an obtained equation with (10.5), we find that $\mu_l\cdot OB=R_{\min}$. Here we should accept $\theta=\theta_{\max}$.

If the eccentricity is set, so, building the straight parallel to the axis *S* on the distance *e* (Fig. 10.15), we will get from the triangle B_3B_1K

$$\mathrm{tg}\theta=\frac{\frac{ds}{d\varphi}-e}{s+\sqrt{B_3O^2-e^2}}.$$

Thus, if $\theta=\theta_{\max}$ $\mu_l\cdot B_3O=R_{\min}$.

In both cases $\mu_l=\mu_S=\mu_{\frac{dS}{d\varphi}}$.

Fig. 10.15 shows that if the angles are equal $\theta = \theta_{max}$ prime radius of a cam with in-line follower is more than with offset follower.

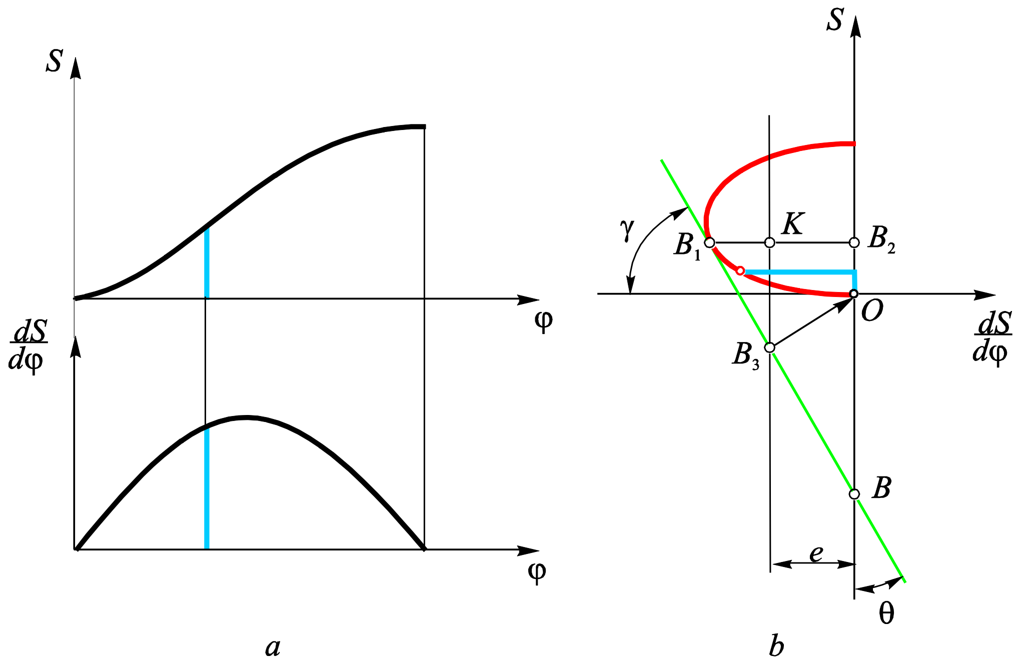


Fig. 10.15. Prime radius determination: *a* – kinematic diagrams;
b – a cyclogram

The eccentricity is used in a cam mechanism with an offset translating follower decreases the dimensions of a cam.

At the form closure a prime radius of a cam is chosen in such a way that an origin of a radius-vector $\vec{R}_{min}$ is situated in the shaded area (Fig. 10.16).

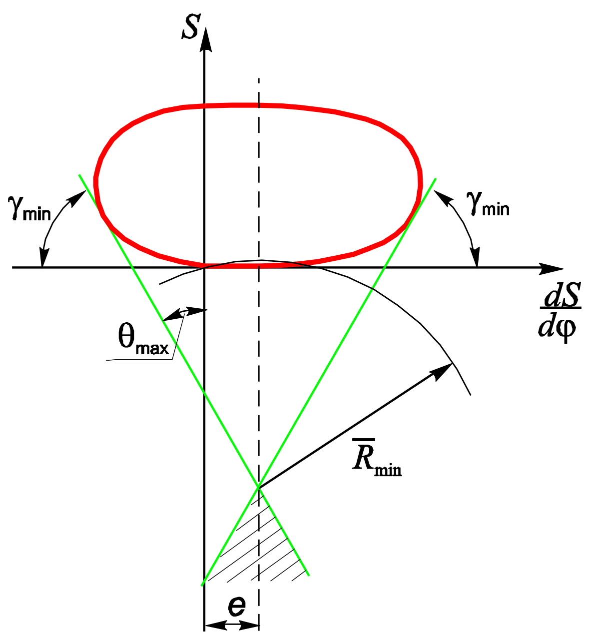


Fig. 10.16. Prime radius determination at the form closure of a cam

As well as for the mechanism with force closure, the use of eccentricity in this case also decreases the dimensions of a cam.

10.6.2.2. Cam mechanism with swinging follower

For this type of cam mechanisms, as well as for mechanisms with translating follower, minimal dimensions of a cam are determined by permissible pressure angles in a higher kinematic pair (Fig. 10.17).

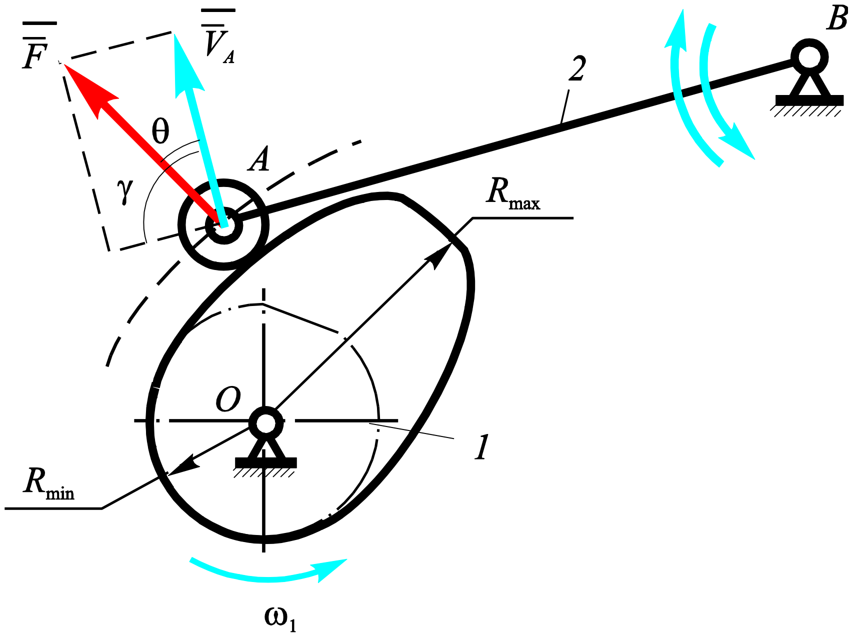


Fig. 10.17. Cam mechanism with swinging follower

Recommended permissible pressure angle and motion transfer angle for swinging follower:

$$\theta_{max} = 45^{\circ};$$

$$\gamma_{min} = 45^{\circ}.$$

The procedure of a determination of a prime radius of a cam does not much change from one depicted above. The peculiarity is that the follower motion cyclogram is built in polar coordinates.

The law of rocker motion, its length and the angle of rocker oscillation ψ_{max} and permissible pressure angles θ_{max} or motion transfer angles γ_{min} should be set.

Determination of prime radius of a cam is carried out in the following order.

1. We construct the follower motion cyclogram, excluding from follower motion diagrams $\psi - \varphi$ and $\dfrac{d\psi}{d\varphi} - \varphi$ the parameter φ. For this we sector the path of point A of a rocker, which is an arc of a circle with radius l_{BA} (Fig. 10.18) in compliance with the diagram $\psi - \varphi$. Through dividing points A_1, A_2, A_3, . . . we build rocker positions and lay off segments $A_iC_i = \left(l_{BA} \cdot \left| ii^* \right| \mu_{\frac{d\psi}{d\varphi}} \right) \dfrac{1}{\mu_l}$. Here μ_l is a scale of the rocker construction BA. We should mention that these segments are built from the point A_i to the centre B of rocker rotation, if directions of rocker and cam rotations coincide. If rotation directions are opposite, we should build segments on the continuation of BA.

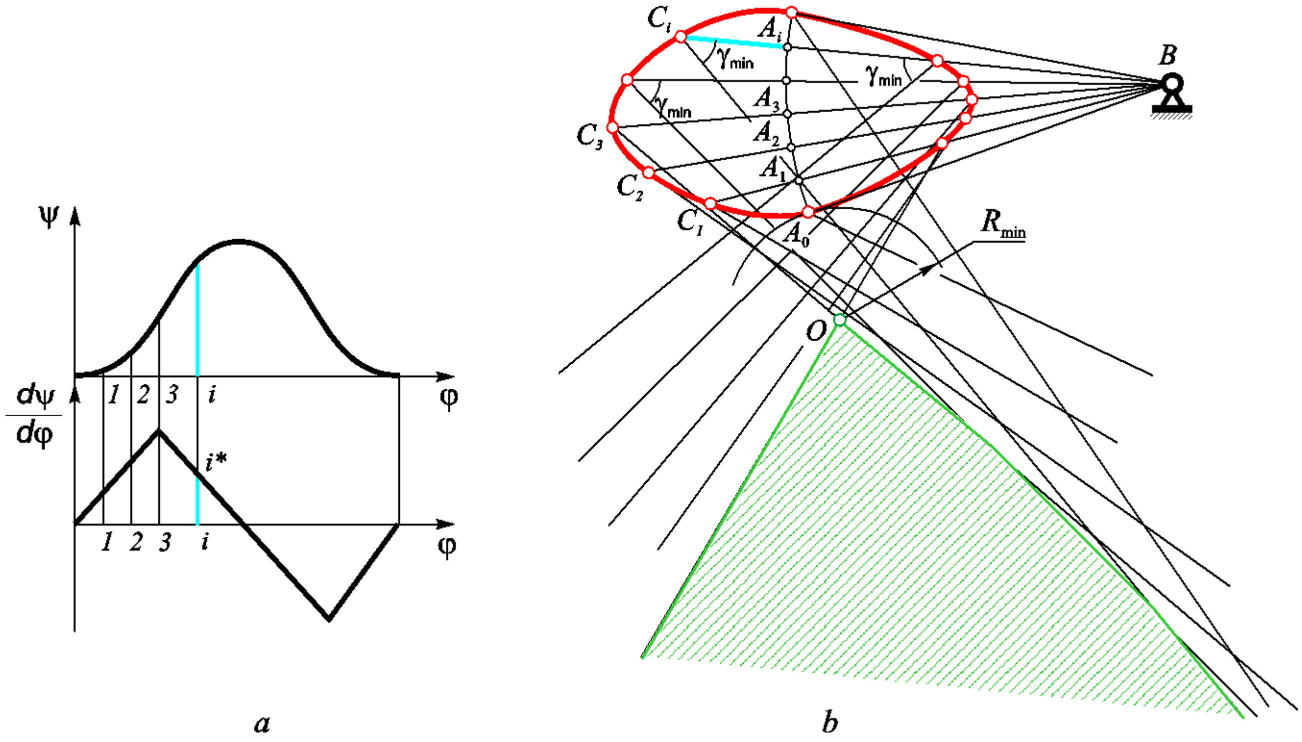


Fig. 10.18. Prime radius determination: *a* – kinematic diagrams; *b* – a cyclogram

2. Through the obtained points C_i we build a closed curve, which is the rocker motion cyclogram.

3. At the points C_i on the sides BC_i we build angles $\gamma_{\min}$.

4. Then we mark the area, which is limited by the sides of built angles that correspond to rising and returning of the rocker (in the Fig. 10.18 this area is shaded). In this area any point can be the center of rotation of the cam. The minimal value of its prime radius will be when the point *O* will be chosen as the center of rotation.

10.6.2.3. Cam mechanism with flat-faced follower

The main condition at determination of prime radius of a cam for such mechanism is assurance of a convex of its profile, as there should be the only one contact point between the follower and the cam.

Let us show how to realize this condition in practice.

For the mechanism, shown in the Fig. 10.19, *a*, we construct an acceleration diagram (Fig. 10.19, *b*). For this purpose we use the scheme of equivalent mechanism delineated by the dot line.

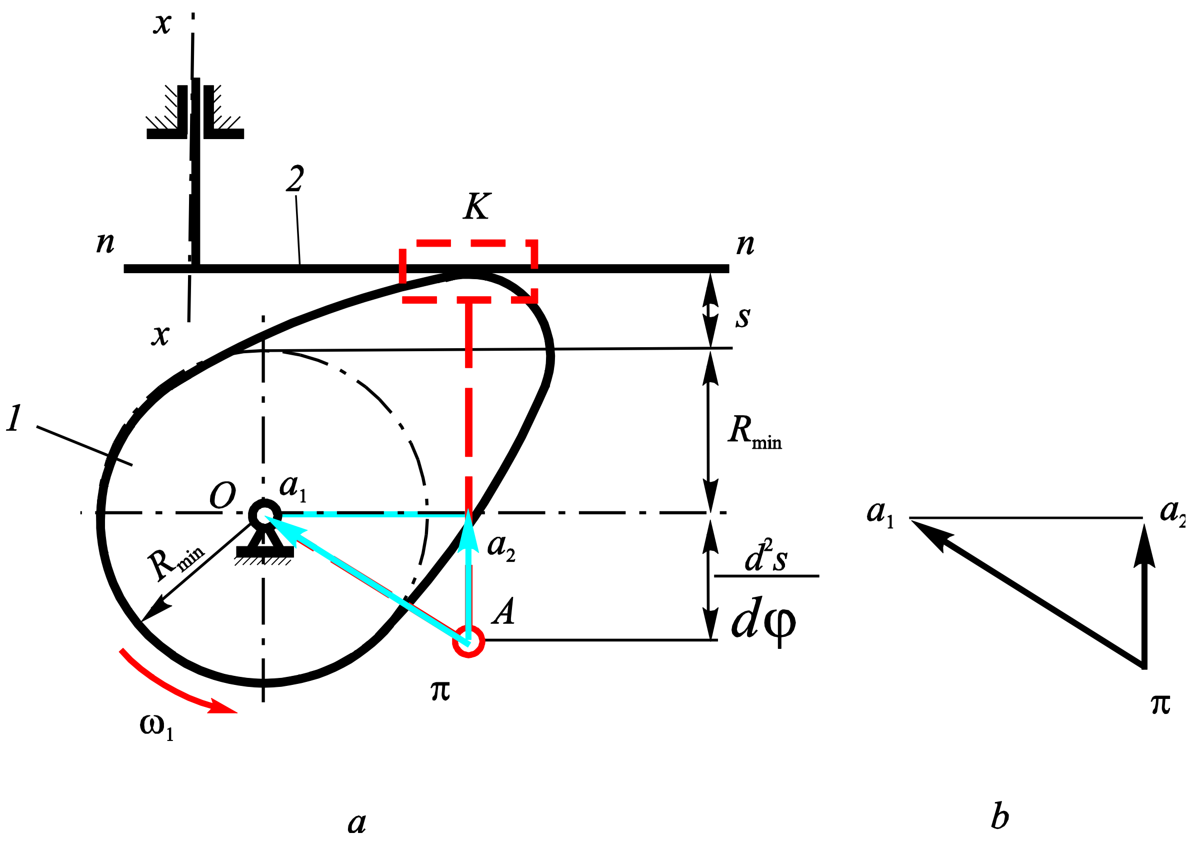


Fig. 10.19. Cam with flat-faced follower: *a* – kinematic scheme;
b – acceleration diagram

Let us superpose the pole of acceleration diagram π with the point A of equivalent mechanism, and a point a_1 – with a point O. The scale of acceleration diagram superposed with kinematic scheme:

$$\mu_a = \frac{a_{A_1}}{\pi a_1} = \frac{\omega_1^2 l_{OA} \cdot \mu_l}{l_{OA}}.$$

Thus

$$\mu_a = \omega_1^2 \mu_l .$$

Then

$$\pi a_2 = \frac{a_{A_2}}{\mu_a} = \frac{a_{A_2}}{\omega_1^2 \mu_l}.$$

Considering that under condition of uniform motion of a cam ($\varepsilon_1 = 0$)

$$a_{A_2} = \frac{d^2 S}{dt^2} = \frac{d^2 S}{d\varphi^2}\omega^2,$$

we write down:

$$\pi a_2 = \frac{d^2 S}{d\varphi^2}\frac{1}{\mu_l}.$$

That is the acceleration diagram superposed with the kinematic scheme (Fig. 10.19), can be considered as an analogue acceleration diagram constructed in the same scale as the kinematic scheme of the mechanism. Then each segment of the analogue acceleration diagram can be summed with any segments of a kinematic scheme.

Convex camber condition of a profile in cam-to-follower contact point is

$$AK > 0.$$

According to the Fig. 10.19 this condition can be written as

$$R_{\min} + S + \frac{d^2S}{d\varphi^2} > 0.$$

Or

$$R_{\min} > -\left(S + \frac{d^2S}{d\varphi^2}\right). \qquad (10.6)$$

The procedure of graphical determination of a prime radius of the cam $R_{\min}$, offered by Ya. L. Heronimus, supposes such order of actions.

1. The cyclogram is constructed by set follower motion diagrams $S - \varphi$ and $\frac{d^2S}{d\varphi^2} - \varphi$ in coordinates $S - \frac{d^2S}{d\varphi^2}$ (Fig. 10.20). Besides, scales $\mu_S = \mu_{\frac{d^2S}{d\varphi^2}}$.

2. From the formula (10.6) we have

$$\frac{-\dfrac{d^2S}{d\varphi^2}}{R_{\min} + S} < 1.$$

It is fully conformed to the design shown in Fig. 10.20: from the negative side of an acceleration analogue of a follower $\frac{d^2S}{d\varphi^2}$ we build tangent to cyclogram angularly 45°.

If $R_{\min} = R_0$ we have condition:

$$R_{\min} + S + \frac{d^2S}{dS^2} = 0, \qquad (10.7)$$

but this is inadmissible from the point of view of contact strength, as according to condition (10.7) we have zero radius of curvature of cam surface in a contact point with a follower. It means that we have break of a cam surface. So the following condition should be met always

$$R_{\min} > R_0.$$

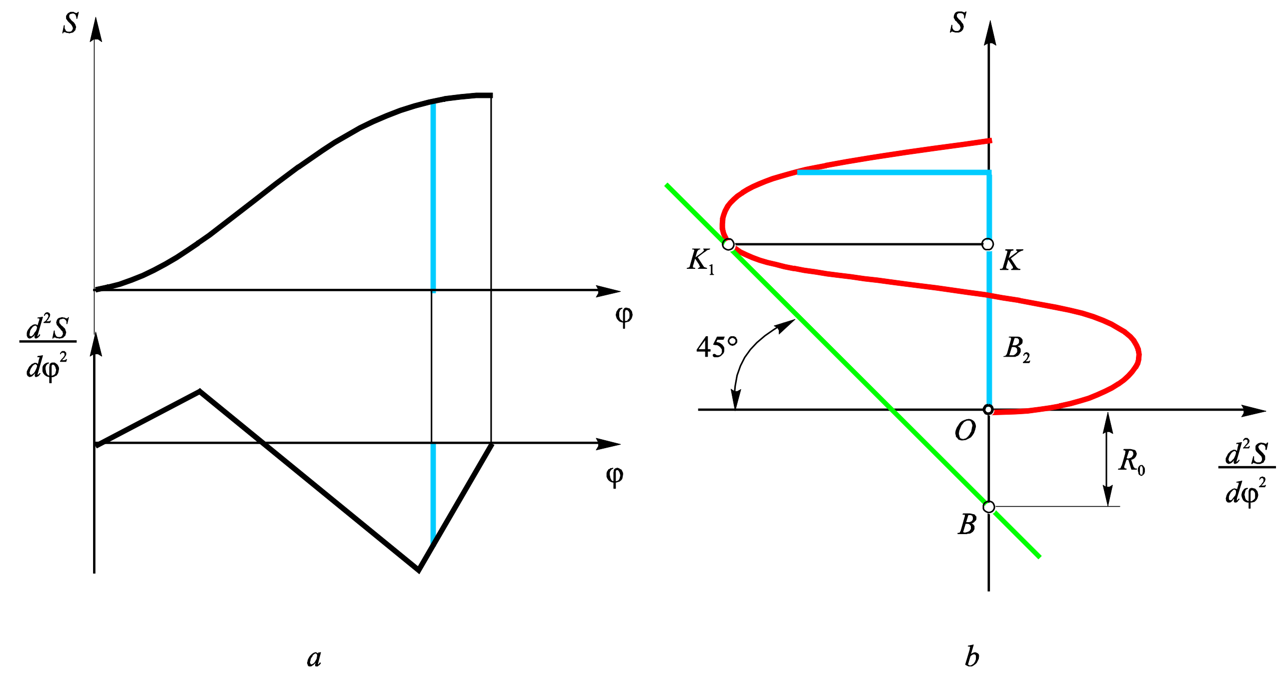


Fig. 10.20. Prime radius determination: *a* – kinematic diagrams; *b* – an cyclogram

3.If as a result of made constructions the crossing point of the tangent and axis *S* is placed higher than the point *O*, it will meet the condition $R_0 < 0$, which does not have any sense. In such case we accept

$$\left|R_{\min}\right| \approx \mu_{\frac{d^2S}{d\varphi^2}} \cdot KK_1 \qquad \left(R_{\min} \approx -\frac{d^2S}{d\varphi^2}\right).$$

10.6.3. Determination of cam surface (kinematic synthesis of a cam)

Kinematic synthesis problem is to determine higher pairing element of a cam, i.a. a cam is shaped according to follower displacements low $S - \varphi$.

Kinematic synthesis is carried out by different methods: analytical and graphical ones. Above we have studied graphical methods of kinematic analysis of certain cam mechanisms. We should admit that the synthesis by graphical methods is carried out in order reversed to analysis [1, 6].

So further we will get to know in detail analytical methods for determining cam profiles.

As well as in the case of kinematic analysis, in kinematic synthesis the inversion principle is used.

10.6.3.1. Cam mechanism with translating follower

Let us study fixed reference system xOy, rigidly bound with the fixed link, origin of which is superposed with a pivot point of a cam (Fig. 10.21). As an initial position we take such position of the system, when the axis y passes through the point A_0 of a start of cam surface rise.

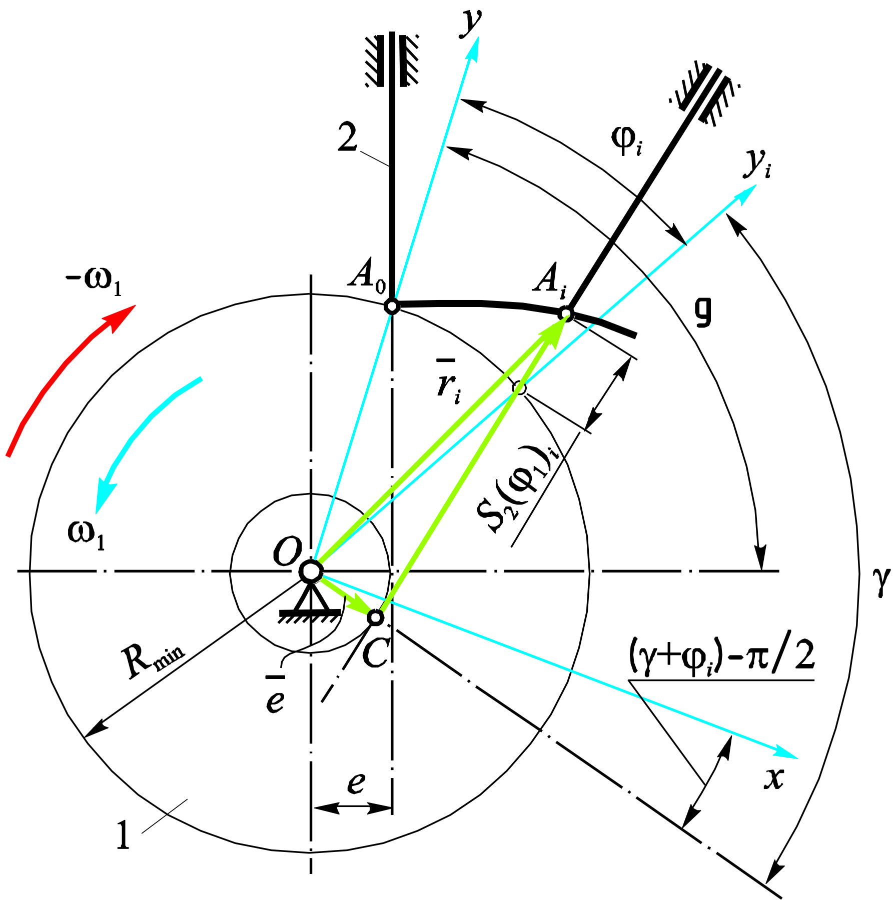


Fig. 10.21. Determination of cam surface (cam mechanism with translating follower)

In compliance with inversion principle, turn of a cam by the angle φ_i is similar to the turn of the fixed link together with a follower relatively to the immovable cam on the same angle in inverse direction (See Fig. 10.21). Here $S_2(\varphi_1)$ is a follower displacement according to the set position function.

Radius-vector of point A_i can be represented as

$$\vec{r}_i = \overrightarrow{OA_i} = \overrightarrow{OC} + \overrightarrow{CA_i}.$$

We mark $\overrightarrow{OC} = \vec{e}$; $\overrightarrow{CA_i} = \overrightarrow{S_2(\varphi_1) + \sqrt{R_{\min}^2 - e^2}}$.

So to say

$$\vec{r}_i = \vec{e} + \overrightarrow{S_2(\varphi_1) + \sqrt{R_{\min}^2 - e^2}}.$$

Then the coordinates of this point:

$$\begin{cases} x_{A_i} = e\cos\left((\gamma + \varphi_i) - \dfrac{\pi}{2}\right) + \left[\sqrt{R_{\min}^2 - e^2} + S_2(\varphi_1)_i\right] \cdot \sin\left((\gamma + \varphi_i) - \dfrac{\pi}{2}\right); \\ y_{A_i} = e\sin\left((\gamma + \varphi_i) - \dfrac{\pi}{2}\right) + \left[\sqrt{R_{\min}^2 - e^2} + S_2(\varphi_1)_i\right] \cdot \cos\left((\gamma + \varphi_i) - \dfrac{\pi}{2}\right). \end{cases}$$

10.6.3.2. Cam mechanism with swinging follower

For a mechanism with swinging follower the turn of a cam by the angle φ_i is similar to the turn on the same angle of a straight OO_1 (Fig. 10.22). The point O_1 moves in the point O_1^*.

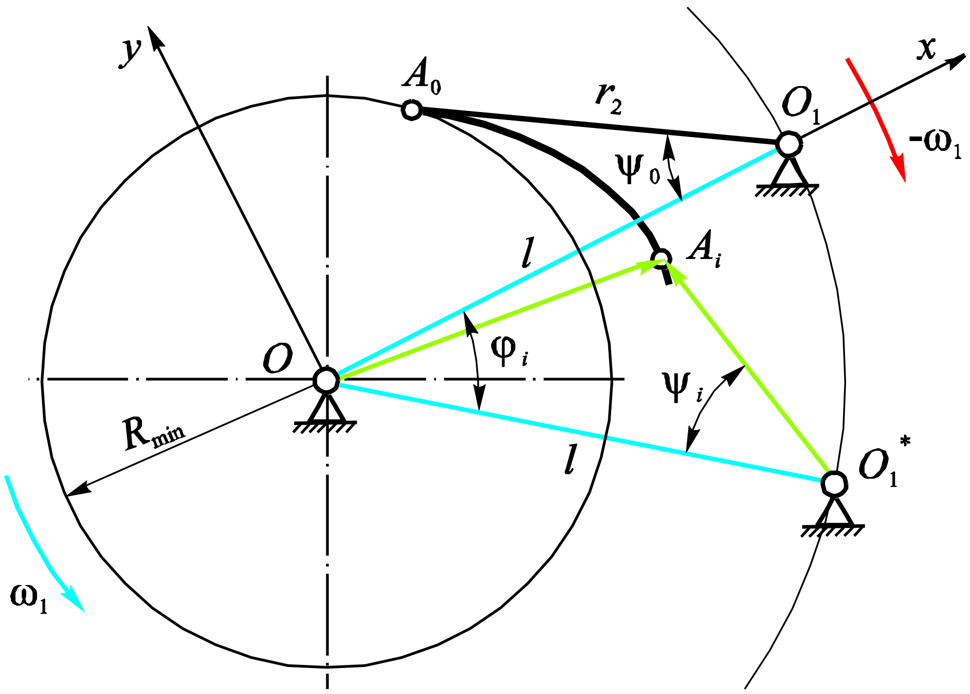


Fig. 10.22. Determination of cam surface (cam mechanism with swinging follower)

We may write down:

$$\overrightarrow{OA} = \overrightarrow{OO_1} + \overrightarrow{O_1^*A}.$$

Here $\left|\overrightarrow{OO_1}\right| = l$; $\left|\overrightarrow{O_1^*A}\right| = r_2$.

In projections onto coordinate axes we have:

$$\begin{cases} x_A = l\cos\varphi_i - r_2\cos(\varphi_i + \psi_i); \\ y_A = -l\sin\varphi_i + r_2\sin(\varphi_i + \psi_i). \end{cases}$$

10.6.3.3. Cam mechanism with flat-faced translating follower

According to the Fig. 10.23 radius-vector of point A can be expressed as:

$$\vec{r}_A = \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}.$$

Here $\left|\overrightarrow{OB}\right| = R_{\min} + S_2(\varphi_1)_i$.

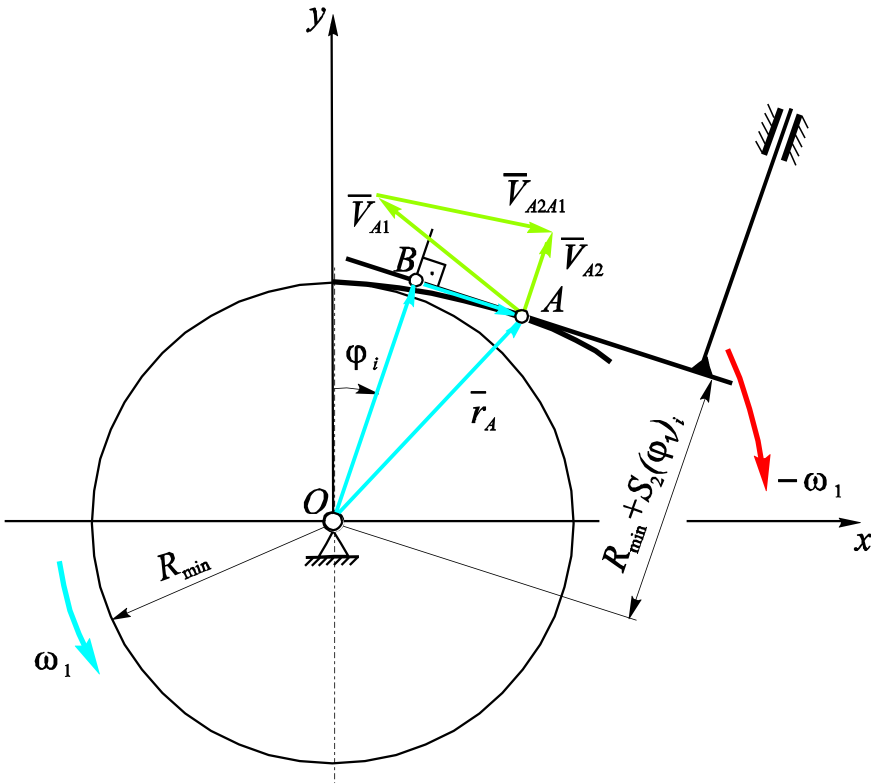


Fig. 10.23. Determination of cam surface (cam mechanism with flat-faced translating follower)

Triangle *OBA* is similar to velocity triangle $\vec{V}_{A_2} = \vec{V}_{A_1} + \vec{V}_{A_2A_1}$. Hence:

$$\frac{V_{A_1}}{V_{A_2}} = \frac{OA}{AB};$$

$$AB = OA\frac{V_{A_2}}{V_{A_1}} = r_A \frac{\frac{dS_2}{d\varphi_1}\omega_1}{\omega_1 r_A} = \frac{dS_2}{d\varphi_1}.$$

Thus,

$$\vec{r}_A = \overrightarrow{\left[R_{\min} + S_2\left(\varphi_1\right)\right]} + \overrightarrow{\left(\frac{dS_2}{d\varphi_1}\right)}.$$

Contact point coordinates of a cam with a face of follower at a set angle φ_i are defined as:

$$\begin{cases} x_A = \left[R_{\min} + S_2\left(\varphi_1\right)\right]\sin\varphi_1 + \left(\dfrac{dS_2}{d\varphi_1}\right)\cos\varphi_1; \\ y_A = \left[R_{\min} + S_2\left(\varphi_1\right)\right]\cos\varphi_1 - \left(\dfrac{dS_2}{d\varphi_1}\right)\sin\varphi_1. \end{cases}$$

10.6.3. Barrel cam design

As an example let us consider barrel cam design with translating follower moved in a direction parallel to the cam axis (Fig. 10.24).

Dynamic synthesis of such mechanism is reduced to the determination of minimum radius of a barrel cam. Its magnitude is defined from the condition of the absence of seizure of a mechanism, i.a. by meeting the condition $\theta \le [\theta]$.

Pressure angle θ is the angle between velocity vector $\vec{V}_2$ and normal line *n-n* (Fig. 10.24).

Let us study velocity triangle, acute angles of which are expressed as

$$\chi_1 = \theta;\ \chi_2 = 90^\circ - \chi_1.$$

According to the cosine law

$$\frac{V_1}{\sin\chi_2}=\frac{V_2}{\sin\chi_1} \quad \text{or} \quad \frac{\omega_1 r_1}{\cos\theta}=\frac{V_2}{\sin\theta}.$$

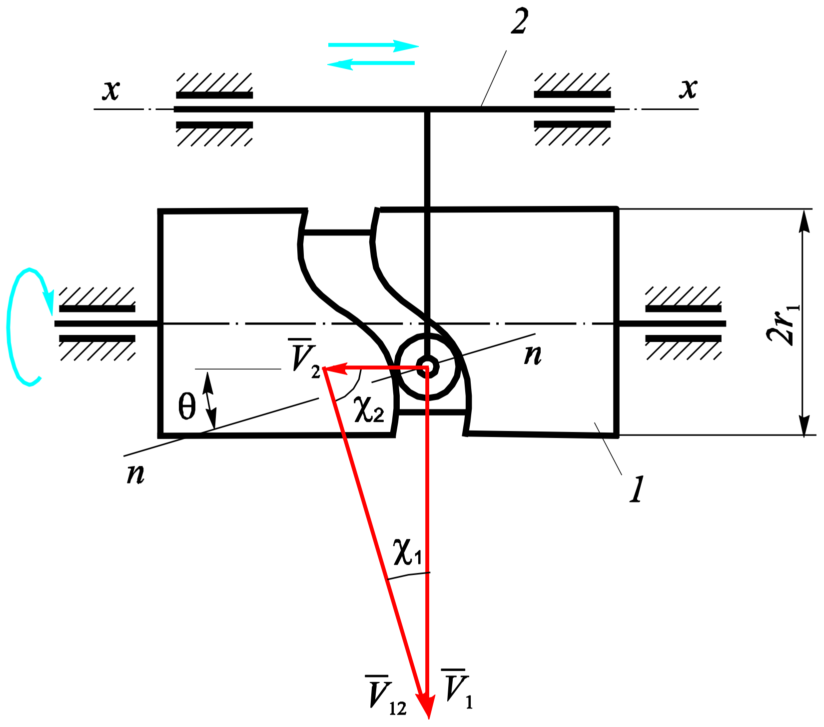


Fig. 10.24. Barrel cam mechanism

Hence

$$r_1=\frac{V_2\cos\theta}{\omega_1\cdot\sin\theta}=\frac{\frac{dS_2}{d\varphi_1}\cdot\omega_1}{\omega_1\cdot\mathrm{tg}\theta};$$

$$r_1=\frac{dS_2}{d\varphi_1}\mathrm{ctg}\theta.$$

Here $\frac{dS_2}{d\varphi_1}=\frac{dS_2}{d\varphi_1}(\varphi_1)$ is transfer function of follower velocity analogues.

Taking the magnitude $[\theta]$, we define the magnitude r_1 for a sequence of successive values of generalized coordinate φ_1 within a cycle of mechanism motion and build the diagram $r_1 = r_1(\varphi_1)$. By this diagram we choose the biggest magnitude of radius r_1 as a minimum radius of a barrel cam.

Graphical determination of a cam profile is executed using cam development on a plane (Fig. 10.25).

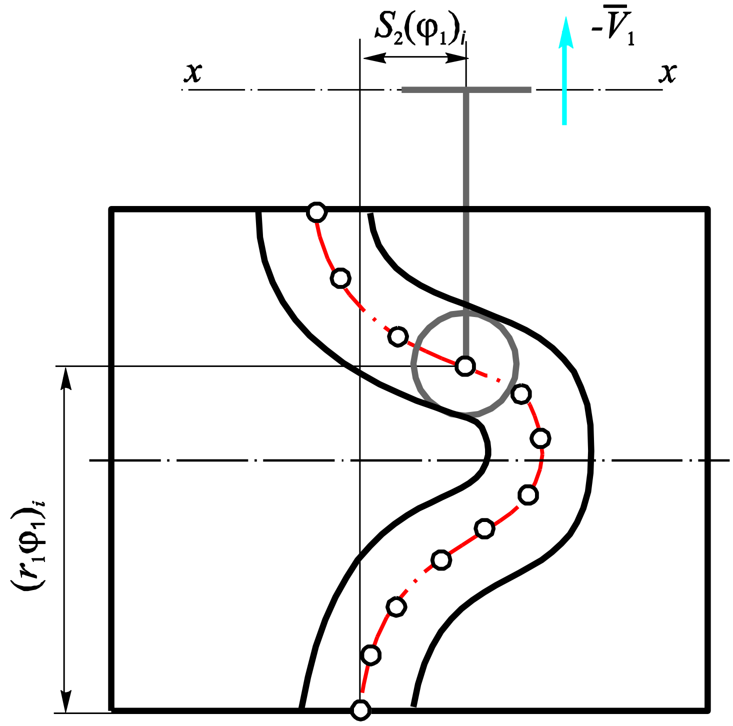


Fig. 10.25. The cam development on a plane

Using the inverse principle, the cam development is considered immovable, and follower axis $x-x$ as one that moves with the velocity $\vec{V}_{x-x} = -\vec{V}_1$ (here $V_1 = \omega_1 r_1$). Follower displacements, in accordance to set position function, are laid off in the direction of the axis $x-x$ (See Fig. 10.25).

In the same way we shape cams for mechanisms with swinging follower [2].

QUESTIONS FOR SELF-TESTING

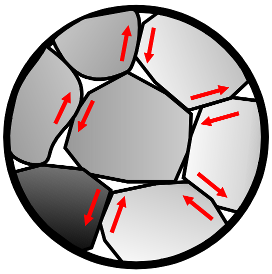
1. What are cam mechanisms?
2. What is a cam mechanism used for?
3. What are the different types of cam mechanisms by kinematic and structural features?
4. List the main advantages and disadvantages of cam mechanisms.
5. In what ways is cam-follower kinematic pair closed in cam mechanisms?
6. What is a face cam and what are the main disadvantages of mechanisms with such cams?
7. A follower of a cam mechanism can be equipped with a roller. For what purpose?
8. What profile of a cam do we call the theoretical profile and what is the acting one?
9. List the phases of follower's motion per cycle of cam mechanism operation.
10. What phases of the follower's motion correspond to the working angle of the cam turning?
11. On what principle are the methods of kinematic analysis of cam mechanisms based? What is the essence of this principle?
12. Under what conditions do "soft" and "hard" impacts occur in the cam-follower kinematic pair?
13. What is the task of synthesis of the cam mechanism?
14. Formulate the base tasks of dynamic and kinematic synthesis of cam mechanisms.
15. What condition should be met when choosing a prime (minimum) radius of cams for mechanisms with translating follower and with swinging one?
16. From what considerations is the allowable value of the pressure angle in the cam mechanism chosen?
17. What is the purpose of designing cam mechanisms with an eccentricity of the axis of follower movement (offset follower)?
18. What condition should be met when choosing a prime (minimum) radius of a cam for a mechanism with flat-faced follower?
19. In what coordinates are the cyclorams of follower motion built during the dynamic synthesis of such cam mechanisms: with a translating follower, with a swinging follower and with a flat-faced pusher follower?

Chapter 11. FRICTION AND WEAR-OUT IN KINEMATIC PAIRS

During the work of machines and mechanisms dissipation of mechanical energy takes place. Friction is the reason of such dissipation.

There are internal and external friction.

Internal friction is a friction between particles of material during its deformation (Fig. 11.1, *a*). The demonstration of this type of friction is that a metal has elastic hysteresis loop (Fig. 11.1, *b*). The wider the loop, the higher are damping characteristics of material that is its ability to damp vibrations (natural oscillations of the real materials damp even in vacuum).



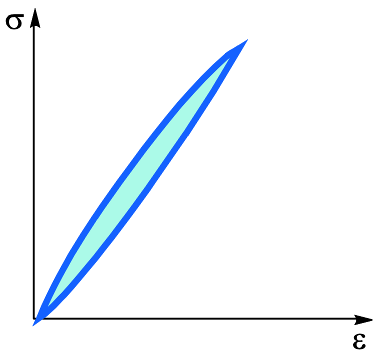


Fig. 11.1. The internal friction: the friction between particles of material (*a*) and the elastic hysteresis loop (*b*)

External friction is the resistance to the relative displacement of contacting bodies or attempt to cause this displacement. This resistance force is called friction force.

Henceforth we will consider only external friction.

11.1 TYPES OF FRICTION

Physics of the friction isn't completely studied. There are different schools which interpret the nature of friction from different sides, for example, from the point of view of physics of metals, electrical nature etc.

Let's consider different types of friction, without deepening in the nature of this phenomenon.

11.1.1. Classification of friction by kinematic characteristic

We differentiate:

- Sliding or kinetic friction (Fig. 11.2, *a*);
- rolling friction (Fig. 11.2, *b*);
- pivoting friction (Fig. 11.2, *c*);
- rolling friction with slippage (in toothing, for example);
- friction by vibratory displacements.

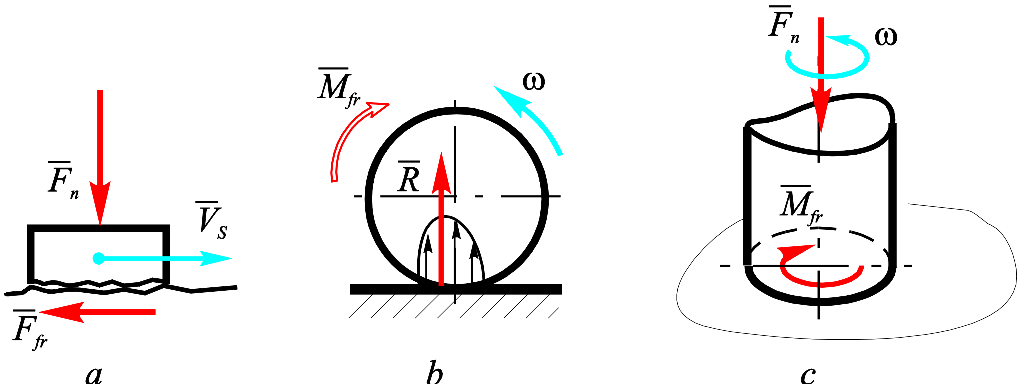


Fig. 11.2. Types of friction by kinematic characteristic: *a* – sliding friction; *b* – rolling friction; *c* – pivoting friction.

11.1.2. Classification of friction by surface condition

We differentiate:

- unlubricated friction – *dry* or *Coulomb friction*;
- friction under lubrication – *fluid friction*.

11.1.3. Static friction and dynamic friction

Static friction aka *stiction* goes before *dynamic friction* aka *kinetic friction*.

Static friction force is always greater than dynamic friction force.

Static friction force, any exceeding of which will set going, is called *the greatest static friction force.*

Friction can be both helpful and harmful.

In general in techniques static friction plays a positive role. Due to this type of friction power transmissions can work, transport can move on the surface and actually we can walk.

As a rule dynamic friction is harmful. It appears under relative displacement of links, and is a cause of wear-out of kinematic pairing elements, power waste etc.

11.2. DRY FRICTION

This friction takes place when there is no lubricant on the friction surfaces (surfaces free from liquids or films covering them).

11.2.1. The Amonton-Coulomb Law

The first systematic studies on friction were conducted by Leonardo da Vinci. We can assume that it was he who introduced the concept of *coefficient of friction*. Later, Amonton repeated his experiments with greater care and confirmed Leonardo's conclusions that friction does not depend on the contact area of interacting bodies, but is determined by their physical nature.

For the first time, Leonard Euler formulated the law of dry friction in a modern interpretation. In turn, Coulomb, continuing the work of his predecessors in this field, filled the dry friction law with physical content suitable for use in engineering practice. Today, the law of dry friction is called the Coulomb's law of friction or the law of Amonton-Coulomb.

Guillaume Amontons (1663−1705)
French physicist who made a significant contribution to the development of mechanics, thermodynamics, molecular physics. Being almost deaf from birth, he had no opportunity to study at the university, so he studied mathematics, physics, geodesy, architecture and other sciences independently. This did not prevent him from becoming a member of the French Academy of Sciences in 1699. In particular, he studied the properties of friction of solids, established the thermodynamic point – the boiling point of water, and also he came out with the idea of the existence of an absolute zero temperature, developed later by Lord Kelvin

Charles-Augustin de Coulomb (1736−1806)
Prominent French physicist, military engineer, member of the Paris Academy of Sciences. He made a great contribution to the development of mechanics and the doctrine of electromagnetism. He formulated a number of fundamental laws and theories, named after him, which formed the basis of modern science: one of the classical strength theories using the maximum shear stress criterion, the law of dry friction, the law of interaction of electric charges and magnetic poles. His name is on the list of the most prominent French scientists, located at the base of the Eiffel Tower.

Let us study dry friction in the lower sliding kinematic pair, when a body contacts with a plane (Fig. 11.3). Here F_n are normal force, and F_{fr} is friction force.

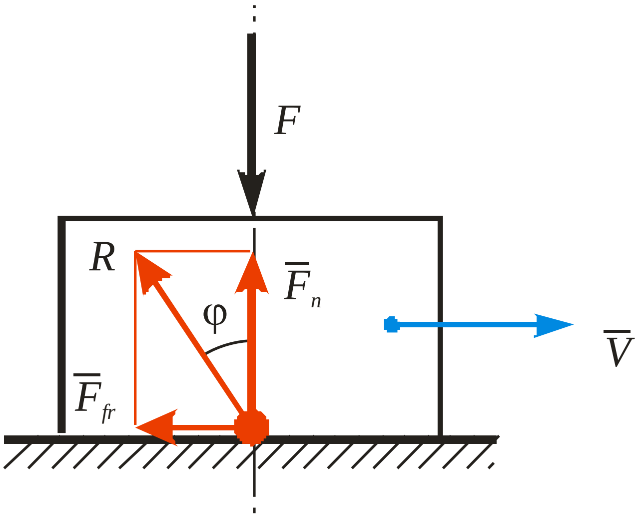


Fig. 11.3. Forces in the lower sliding kinematic pair

Friction force depends on value of normal force in kinematic pair, oppositely directed to the slip velocity and is determined by the coefficient of friction.

$$F_{fr} = f \cdot F_n.$$

Here f is coefficient of friction.

According to Fig. 11.3 we can write down:

$$tg\varphi = \frac{F_{fr}}{F_n} = f\ .$$

That is

$$f = tg\varphi.$$

Here the angle φ is called *the angle of friction*.

The value of a coefficient of friction and an angle of friction for the different pairs of materials are given in reference literature on machinery, physics. For some pairs of materials, the values of the friction coefficients are given in Appendix 8.

We differentiate coefficient of static friction f_{st} and coefficient of dynamic friction f. They are submitted to the condition

$$f < f_{st}.$$

11.2.2. Factors which influence on the coefficient of friction

The coefficient of friction isn't a constant magnitude. It is influenced by a number of design, technological and operational factors:

– the nature of contacting bodies;

– surface state – there is certain optimal surface roughness, under which friction will be minimal (for example, ideally polished surface isn't ideal from the point of view of friction);

– slip velocity of the interacting bodies.

Let us consider the last factor in more detailed.

According to the Amonton-Coulomb law, the coefficient of friction does not depend on slip velocity (Fig. 11.4, *a*). If $F_n = Const$ and $F_{fr} = Const$.

In practice dry friction depends on slip velocity (Fig. 11.4, *b*).

There is an optimal value of slip velocity V_{top} (*top speed*), at which friction and a coefficient of friction are minimal. In low slip velocity zone there is sharp friction decrease when the velocity is enhancing. According to such dependence of friction on sliding velocity under low traverse speed there is unstable (stick-slip) motion. Such effect is typical for processing equipment. It accompanies by the movement judder, the high wear of slideways and tool. Additional dynamic loads appear in the mechanisms, the accuracy of equipment functioning falls.

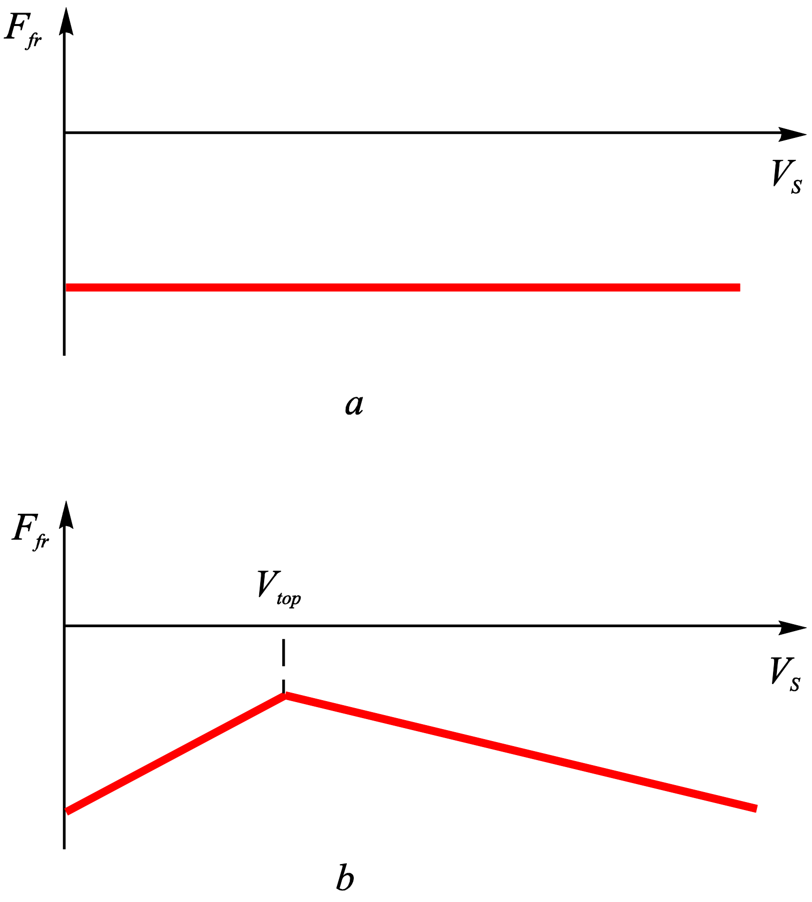


Fig. 11.4. Friction force – slip velocity dependence:
a – according to the Amonton- Coulomb law;
b – in practice

To eliminate this effect we can replace dry friction by fluid friction with special hydrostatic lubrication. Instead of traditional cast iron and bronze for slideways and sliders teflon is used, where it is possible. Sliding friction is replaced by rolling friction.

11.2.3. Friction in lower kinematic pairs

Except for listed factors which have an influence on the coefficient of friction value, the form and relative positions of pairing elements are also important. For their different types so-called *superficial coefficient of friction* is determined. Let us consider a sliding kinematic pair, formed by links 1 and 2 (Fig. 11.5), that contact on the arbitrary shape surface.

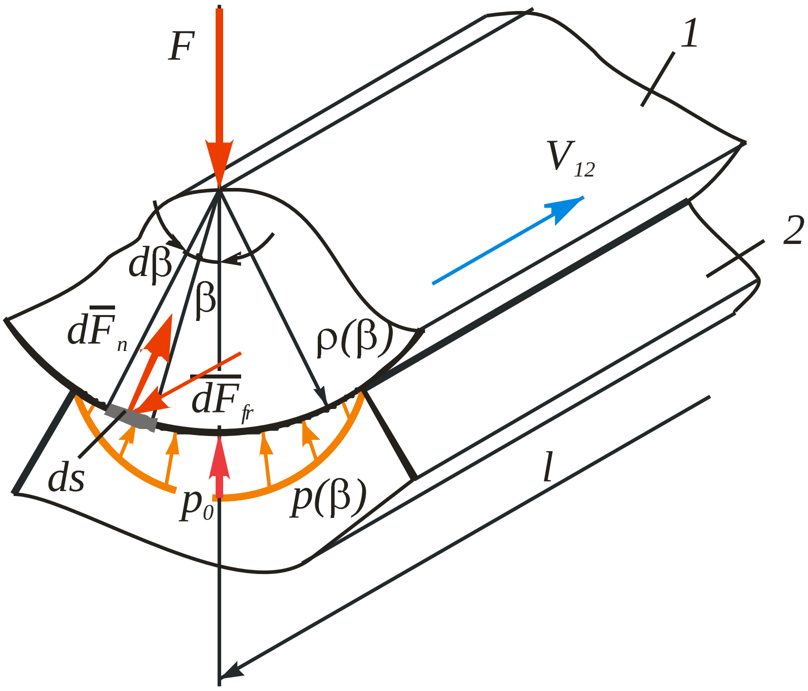


Fig. 11.5. Common sliding kinematic pair

Here l is the length of contact surface, $\rho(\beta)$ – radius of curvature of the contact surface in arbitrary point. Let us separate out the surfent of contacting area $l \cdot ds$. Friction on it is

$$dF_{fr} = fdF_n;$$

$$dF_n = p(\beta)lds = p(\beta)l\rho(\beta)d\beta.$$

That is

$$dF_{fr} = fp(\beta)l\rho(\beta)d\beta.$$

Resulting force of friction:

$$F_{fr} = fl\int_{-\beta_1}^{\beta_2} p(\beta)\cdot\rho(\beta)d\beta. \qquad (11.1)$$

11.2.3.1. Friction in wedge-shaped slider

The design scheme of such slider is shown in Fig. 11.6.

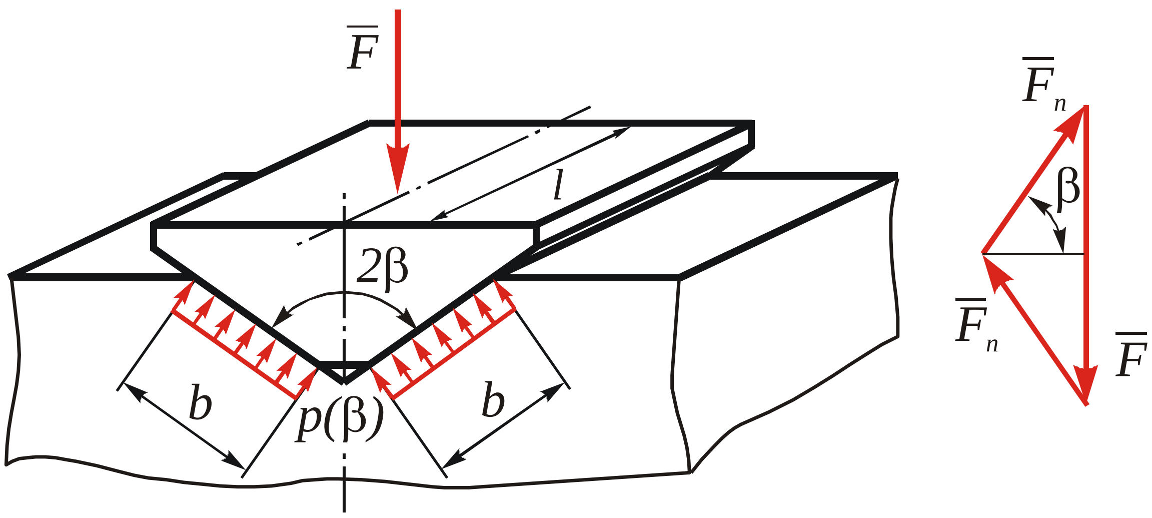


Fig. 11.6. Forces acted in a wedge-shaped slider

From the force diagram we have:

$$2F_n \sin\beta = F\ .$$

Then a pressure on the contact surface is

$$p(\beta) = \frac{F}{2lb\sin\beta}\ .$$

Taking into account that in our example

$$\int_{-\beta_1}^{\beta_2} \rho(\beta)\, d\beta = 2b\ ,$$

according to (11.1) we'll have:

$$F_{fr} = fl\frac{F}{2lb\sin\beta}\cdot 2b = f'\cdot F\ .$$

Here superficial coefficient of friction for a wedge-shaped slider is

$$f' = \frac{f}{\sin\beta}\ .$$

11.2.3.2. Friction in sliding pair with cylindrical slider

In the cylindrical slider (Fig. 11.7) the radius of curvature of the contact surface $\rho(\beta) = r = Const$.

Sliding pair with unrun contact surfaces. In this case we accept uniform pressure distribution $p(\beta) = p = Const$ (Fig. 11.7).

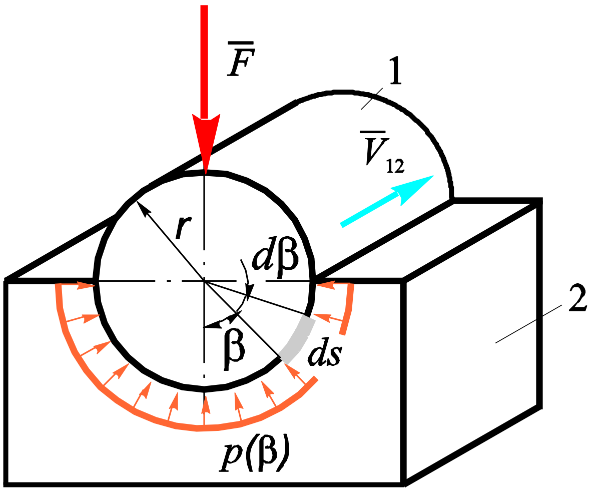


Fig. 11.7. Cylindrical slider in sliding pair with unrun contact surfaces

Then from equilibrium condition of the slider, if we project the force F_n to the direction of the force F, we will obtain:

$$F = 2\int_{0}^{\pi/2} prl \cdot \cos\beta d\beta = 2prl \,.$$

Hence

$$p = p(\beta) = \frac{F}{2rl}.$$

Putting $p(\beta)$ and $\rho(\beta)$ into (11.1), we will have

$$F_{fr} = fl\int_{-\pi/2}^{\pi/2} \frac{F}{2rl} \cdot rd\beta = fF \cdot \frac{\pi}{2} = f' \cdot F \,.$$

Here superficial coefficient of friction is

$$f' = \frac{\pi}{2} f \,.$$

Run-in sliding pair. In run-in pair cosine pressure distribution law is accepted (Fig. 11.8):

$$p(\beta) = p_0 \cos\beta \,.$$

Then

$$F_{fr} = fl \int_{-\pi/2}^{\pi/2} p_0 \cos\beta \cdot r d\beta = 2 frlp_0 \,. \qquad (11.2)$$

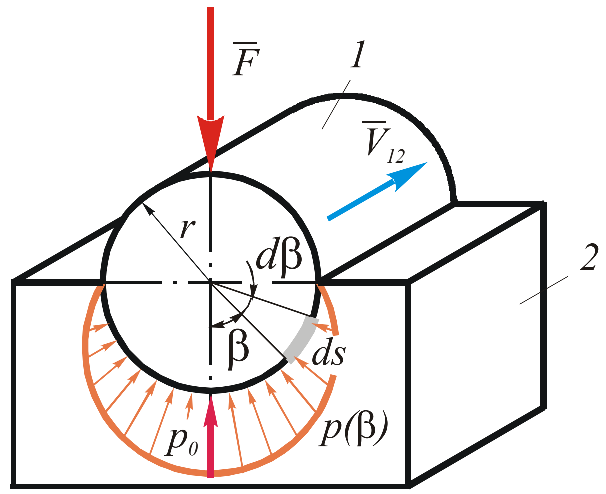


Fig. 11.8. Cylindrical slider in run-in pair

From the equilibrium condition of the link 1:

$$F = \int_{-\pi/2}^{\pi/2} pl \cos\beta ds = \int_{-\pi/2}^{\pi/2} p_0 l \cos^2\beta r d\beta = p_0 lr \int_{-\pi/2}^{\pi/2} \cos^2\beta d\beta = p_0 lr \frac{\pi}{2} \,.$$

Hence

$$p_0 = \frac{2F}{\pi lr} \,.$$

Then from the formula (11.2)

$$F_{fr} = 2\,frl \cdot \frac{2F}{\pi lr} = \frac{4}{\pi} fF = f' \cdot F\,,$$

where superficial coefficient of friction

$$f' = \frac{4}{\pi} f\,.$$

11.2.3.3. Friction in turning pair

In turning kinematic pairs (Fig. 11.9) with elements, made in the form of cylinders and are loaded with the force *F*, pressure distribution is accepted as in cylindrical slider. Force of friction is determined as for that case.

Total force $\overline{F}_R = \overline{F}_{fr} + \overline{F}_n$ (Fig. 11.9) touches the circle with radius $f'r$ that outlines the so-called *friction circle*. The moment of this force relatively to hinge axis hinders to rotation.

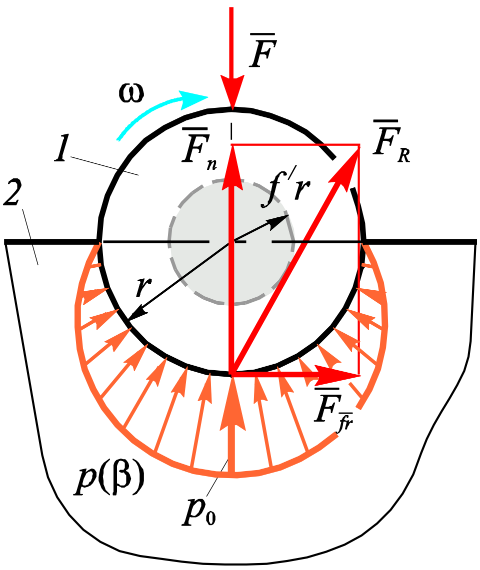


Fig. 11.9. Turning pair

For ball-and-socket hinge (Fig. 11.10, *a*) superficial coefficient of friction is $f' = 1{,}27 f$.

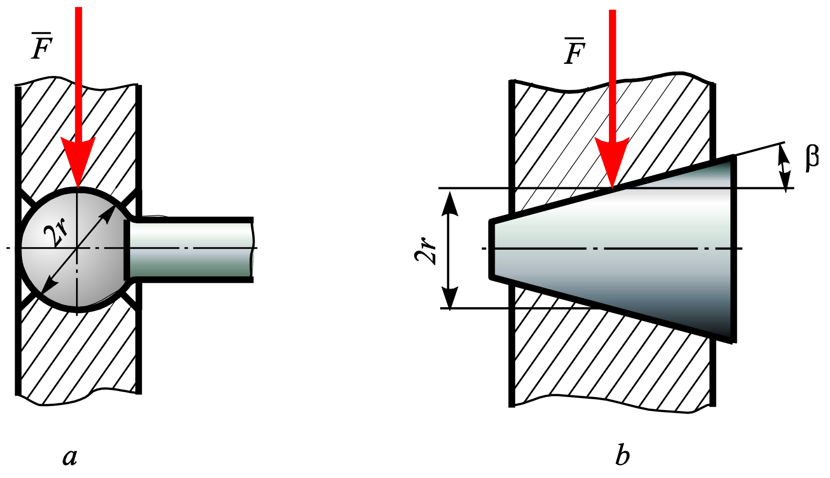


Fig. 11.10. Turning pairs: *a* – ball-and-socket hinge;
b – with conical elements

For pairs with conical elements (Fig. 11.10, *b*) superficial coefficient of friction is $f' = \dfrac{f}{\cos\beta}$.

11.2.3.4. Friction in sliding thrust bearing

The Fig. 11.11 shows a sliding thrust bearing with flat contact surfaces. Here kinematic pair is formed by the abutment 1 and the socket 2, loaded by longitudinal force *F*. In this case pivoting friction appears on the abutment surface, which follows the Amonton-Coulomb law.

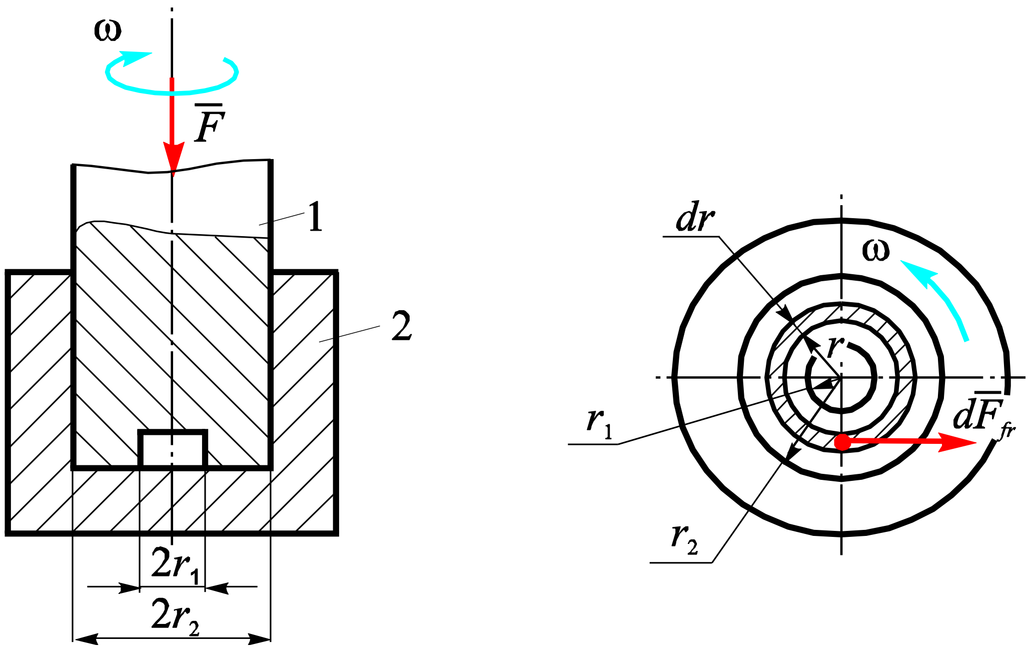


Fig. 11.11. Sliding thrust bearing

If to consider pressure distribution p along the full width of ring as uniform, it can be written down as:

$$p = \frac{F}{\pi\left(r_2^2 - r_1^2\right)}. \tag{11.3}$$

Let us separate a ring with thickness dr. Moment made up by the force dF_{fr} on this area, equals

$$dM_{fr} = dF_{fr} \cdot r;$$

$$dF_{fr} = f dF_n = f \cdot p 2\pi r dr.$$

Hence

$$dM_{fr} = f 2 p \pi r^2 dr.$$

Integrating we will have

$$M_{fr} = \int_{r_1}^{r_2} 2\pi f p r^2 dr = \frac{2}{3}\pi f p\left(r_2^3 - r_1^3\right).$$

Or taking into consideration (11.3)

$$M_{fr} = \frac{2}{3} f F \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}.$$

If the abutment is non-ring, that is $r_1 = 0$, so

$$M_{fr} = \frac{2}{3} F \cdot f \cdot r.$$

11.2.3.5. Friction in screw pair

Let's study a screw-thread (a square thread in our case) is shown in Fig. 11.12.

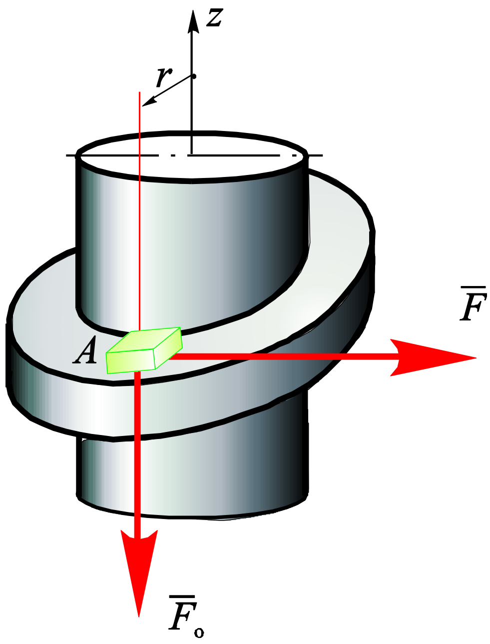


Fig. 11.12. Screw pair

On the nut, represented by element *A*, acts some force F_0 and moment in plane, perpendicular to the axis *z*. This moment is represented by the force *F*:

$$M = F \cdot r .$$

In order to make nut in immobility or in uniform motion condition along the screw, equilibrium conditions of forces acting on nut must be fulfilled.

Let us study the development of one thread on a plane (Fig. 11. 13, *a*).

Equilibrium condition of the element *A*, loaded by forces system, converged to a single point is written down as:

$$\bar{F} + \bar{F}_0 + \bar{F}_R = 0 .$$

From the force diagram (Fig. 11.13, *b*)

$$F = F_0 tg\left(\varphi + \gamma\right). \tag{11.4}$$

Here γ is a lead angle of the thread.

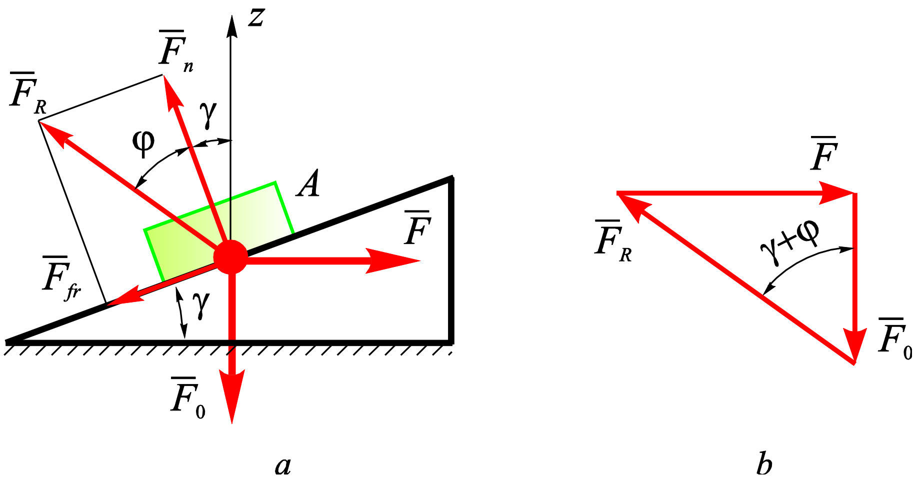


Fig. 11.13. Screw pair: *a* – development of one thread; *b* – force diagram

So the moment, imposed to the nut

$$M = F_0 r tg(\gamma + \varphi).$$

Condition (11.4) obtained for the case, when a nut moves against the direction of the force F_0 (tightening of screw-thread or jack lifting).

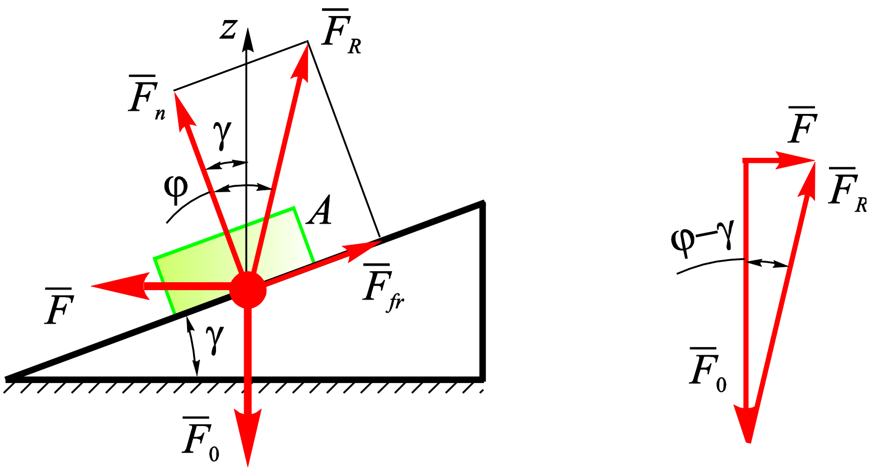
If the nut moves in the direction of the force F_0 (Fig. 11.14), we will have

$$F = F_0 tg(\varphi - \gamma).$$

If $\gamma = \varphi \Rightarrow F = 0$, that is the nut moves uniformly under the force F_0 (without driving moment *M*).

If $\gamma > \varphi$ – we have accelerated motion under the force F_0.

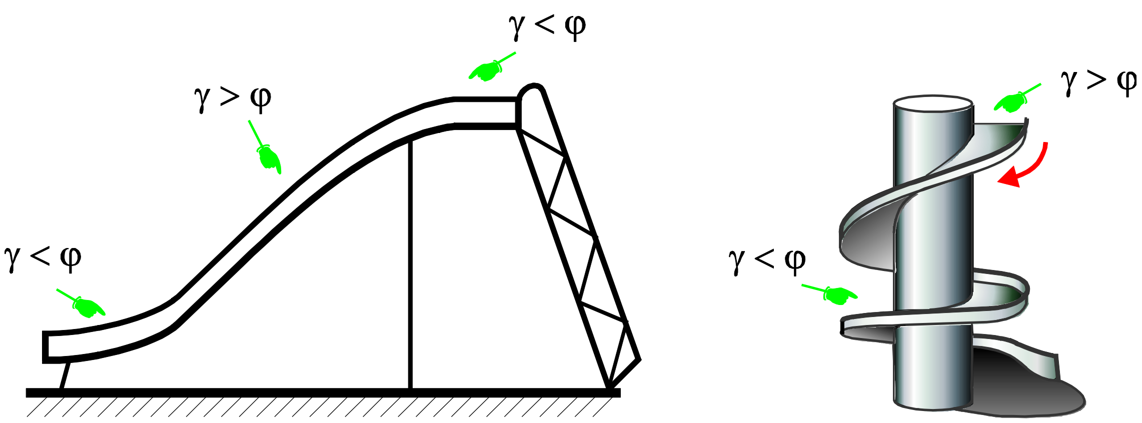
If $\gamma < \varphi$ – we have self-braking condition. Under this condition nut motion is impossible without action of moment $M = F \times r$.



a *b*

Fig. 11.14. Screw pair: *a* – development of one thread; *b* – force diagram

Foregoing cases can be exemplified by chute in playground (Fig. 11.15, *a*), slide for weight transfer from one floor to another (Fig. 11.15, *b*) etc.



a *b*

Fig. 11.15. Real examples of self-braking condition usage:
a – the chute; *b* – the slide

Let us determine friction in the screw. According to the Fig. 7.13:

$$F_{fr} = F_R \sin\varphi .$$

From the force diagram

$$F_R = \frac{F}{\sin(\varphi + \gamma)} .$$

Then

$$F_{fr} = F\frac{\sin\varphi}{\sin(\varphi+\gamma)}.$$

After trigonometric transformation:

$$F_{fr} = F\frac{tg\varphi}{\sin\gamma + tg\varphi\cos\gamma} = F\frac{f}{\sin\gamma + f\cos\gamma}. \qquad (11.5)$$

Equation (11.5) can be used for square thread.

This equation can be also used for determining of friction in triangular thread (Fig. 11.16), if to put superficial coefficient of friction f' instead of coefficient of friction f (for wedge-shaped slider) with apex angle $2(90-\alpha/2)$:

$$f' = \frac{f}{\sin\left(90-\frac{\alpha}{2}\right)} = \frac{f}{\cos\frac{\alpha}{2}}. \qquad (11.6)$$

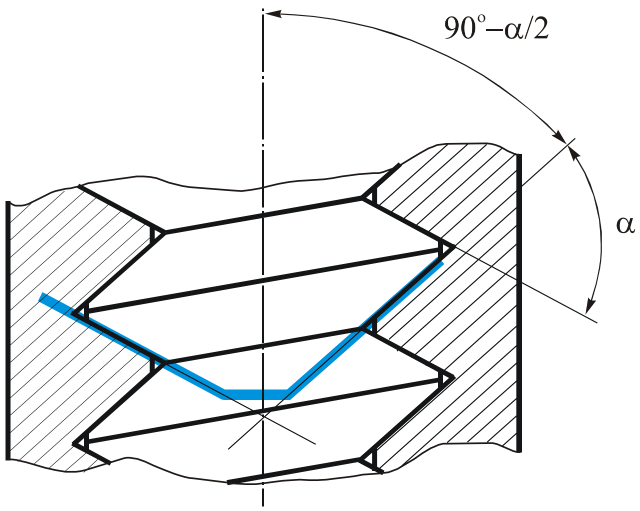


Fig. 11.16. Triangular thread

Then

$$F_{fr} = F\frac{f'}{\sin\gamma + f'\cos\gamma}.$$

Here superficial coefficient of friction f' is determined by the formula (11.6).

11.2.3.6. Friction in kinematic pairs with flexible link

This task was first accomplished by L. Euler.

Let's study a flexible body – a belt thrown over a pulley (Fig. 11.17). To move the belt relative to the pulley, it is necessary that the tension forces in its sides are subject to condition $F_2 > F_1$.

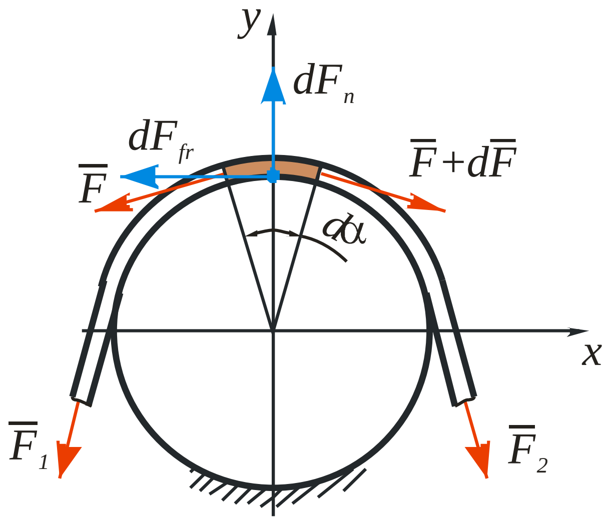


Fig. 11.17. Loading condition of a belt

We separate out the element of a belt within the bounds of angle $d\alpha$. Equilibrium conditions of the projections of forces acting on it onto axes *y* and *x* are:

$$\sum y = dF_n - (F + F + dF)\sin\frac{d\alpha}{2} = 0;$$

$$\sum x = -dF_{fr} - F\cos\frac{d\alpha}{2} + (F + dF)\cos\frac{d\alpha}{2} = 0.$$

From the first equation we can write down:

$$dF_n \cong F d\alpha,$$

and from the second

$$dF_{fr} \cong dF\,. \qquad (11.7)$$

According to the Amonton-Coulomb law

$$dF_{fr} = f'dF_n = f'Fd\alpha .$$

Or

$$\frac{dF_{fr}}{F} = f'd\alpha . \tag{11.8}$$

Taking into account (11.6), we integrate the expression (11.8) within certain limits:

$$\int_{F_1}^{F_2} \frac{dF}{F} = \int_0^{\alpha} f'd\alpha .$$

We will have:

$$\ln\left(\frac{F_2}{F_1}\right) = f'\alpha ,$$

or

$$\frac{F_2}{F_1} = e^{f'\alpha} . \tag{11.9}$$

The equation (11.9) is called *Euler's equation for tensions in belts*.

For the full angle α (it is called *wrapping angle*) subtending the arc of contact between the pulley and the belt we can write down

$$F = F_2 - F_1 = F_{fr} .$$

Then friction force between the belt and the pulley is

$$F_{fr} = F_1\left(e^{f'\alpha} - 1\right).$$

11.3. LUBRICATED FRICTION

Lubricated friction is a case of fluid friction where a lubricant of any type separates two solid surfaces

Lubrication functions:

– to decrease the coefficient of friction;
– to remove heat;
– to prevent corrosion;
– damping of dynamic loads.

Kinds of lubricants:

– solid lubricants;
– lubricating liquid, aka fluid lubricant;
– gas lubricant;
– lubricating grease or dope;
– borderline or thin-film lubricant.

Consider the main characteristics of different types of lubrication.

11.3.1. Solid lubricant

Here rubbing bodies are separated with the help of solid greasing substance. Usually it is powder-like or lubricating graphite. This substance has no specific characteristics.

11.3.2. Lubricating liquid

These are natural and synthetic oil, water.

Let's study the main characteristics of lubricating liquid:

Dynamic and kinematic viscosity. Dynamic viscosity is measured in *poise* and kinematic viscosity is measured in $°E$ (*Engler unit*). In techniques is more often used kinematic viscosity, which is determined by the oil flow speed through calibrated orifice with the diameter near 2,8 mm. Viscosity depends on temperature: $°E100$ – kinematic viscosity at 100ºC; $°E50$ – at 50ºC and so on.

Kinematic viscosity is defined by special tables.

Adherence. This is the ability of lubricant to wet the surface and that means to create adsorption layer on rubbing surfaces.

Almost everybody knows that the water wets a surface worse than oil, and mercury – much worse than water (Fig. 11.18).

Fig. 11.18. Examples of wetting degrees of the surface with liquids

Flash temperature of oil fume. At some temperature a fire appears over the surface of lubricant. For machines oil flash point are about 250…300ºC. There are synthetic lubricants with the flash point more than 600ºC.

Thickening temperature. When the temperature is falling, a lubricant viscosity increases and at some critical temperature it loses its qualities – thickens. Car engine before starting is warmed to certain temperature, for lubricant to obtain the necessary viscosity.

Acid-resistance. That is the ability of lubricants to resist degradation by acids.

There are other specific qualities. You can read about them in special manuals.

11.3.3. Grease

Grease is a semisolid lubricant. It includes, for example, fatty and synthetic solid oil of different types, silicone grease, lithium complex grease etc. By the way, rendered pork fat is one of the best lubricant grease.

Let’s consider main characteristics of viscous lubrication.

Penetration. The cone penetration method is used for grease testing. This method employs a calibrated cone that is dipped into a fixed-size volume of grease for a defined time period (Fig. 11.19). The depth that the cone is able to penetrate the grease is used to rate the grease's consistency with a special scale.

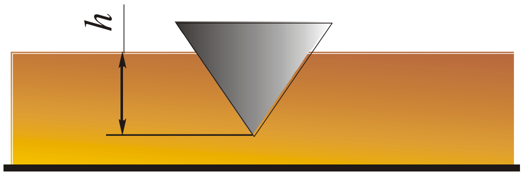


Fig. 11.19. Scheme of cone penetration test

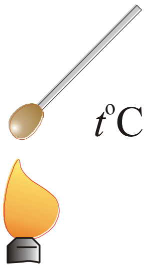


Fig. 11.20. Scheme of drop temperature test

Drop temperature. We take some grease on a stick (Fig. 11.20) and heat it. The temperature, at which the drop falls, is the drop temperature.

Water-resistance. Grease must be insoluble in the water.

11.3.4. Gas lubricant

The function of the lubricant here is performed by gas: air, nitrogen, rare gases. Such kind of lubricant is widely used in kinematic pairs of precision instruments.

11.3.5. Thin-film lubrication

If the thickness of a fluid film is less than 0,0001 mm, lubricant properties differ from bulk properties. That is why friction and wear of such bodies are defined by the properties of contact surfaces and lubricant layer, different from bulk properties.

In Fig. 11.21 there is a section of contact zone of two bodies with hyperfine fluid film.

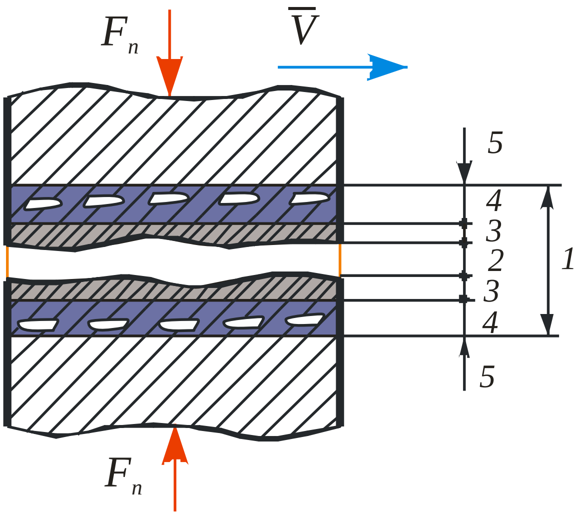


Fig. 11.21. Scheme of the third body

The intermediate layer 1 is called the third body between main materials of friction pair 5. It consists of adsorption lubrication layer 2, oxide film 3 and defective layer of the main material 4.

We differentiate also:

а) *hydrostatic* and *gas-static lubrication*, when a fluid or gas is brought under the external pressure;

б) *hydrodynamic lubrication* and *gas-dynamic lubrication*, when bodies that form friction pair are separated with the help of pressure, self-oscillated in fluid film under the relative motion of these bodies;

в) *elastohydrodynamic lubrication* when characteristics of friction and thickness of fluid film between contact surfaces is defined by elastic properties, creep, relaxation of material.

11.3.6. Sliding friction of lubricated bodies

The founder of hydraulic theory of lubrication is Nicolay Petrov, who published its basic statements in 1883.

N.P. Petrov (1836–1920)
Russian mechanical scientist, professor. He headed the department of Steam Mechanics of St. Petersburg Institute of Technology, and then the department of Railway Affairs. N.P. Petrov made a significant contribution to the development of the theory of friction, wear and grease. He was a member of the Engineering Council of the Society of Russian Railways. In 1884 he was awarded the Lomonosov Prize for his work "Friction in Machines and the Influence of Lubricating Oils on It," where he laid the foundations of the theory of hydrodynamic lubrication. From 1895 to 1905 N.P. Petrov headed the Russian Technical Society. He was elected an honorary member of the St. Petersburg Academy of Sciences.

Fluid friction can be considered as viscous shear between layers of fluid slices, because there is no direct contact between rubbing bodies.

Coefficient of fluid friction f depends on relative velocity of lubricant layers V, normal pressures F_n and coefficient of fluid viscosity μ:

$$f = f\left(V, F_n, \mu\right).$$

Here μ – dynamic viscosity, $N{\cdot}\mathrm{s/m^2}$.

N. Petrov formulated such conditions for fluid friction:

– lubricating fluid keeps in a gap;

– there must be internal pressure in fluid film counteracting external force;

– lubricating fluid separates rubbing surfaces;

– fluid film thickness must be not less than minimal boundary which is defined by asperities of rubbing surfaces.

To perform the first condition adhesion power of fluid with rubbing surface must be more than adhesion power between fluid slices.

To perform the second condition a lubricating fluid must inject spontaneously into the gap between rubbing surfaces. The wedge-shaped liquid film is formed between them and sliding body come floats (Fig. 11.22). This is due to a pressure normal to the sliding direction which is generated in fluid and tends to separate one surface from another. For the journal 1 which lays in a bearing part 2 (Fig. 11.23), appearance of fluid wedge 3 is connected with diameters disparity: diameter of bearing part is bigger than one of a journal. During the journal rotation lubricating fluid injects into a wedge gap. In a fluid film appears a pressure which counteracts external load and shaft surfaces above the fluid film. At high speeds, the axis of journal tends to coincide with the bearing axis.

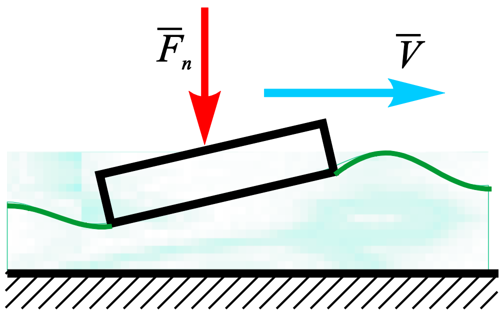


Fig. 11.22. Scheme of fluid wedge formation

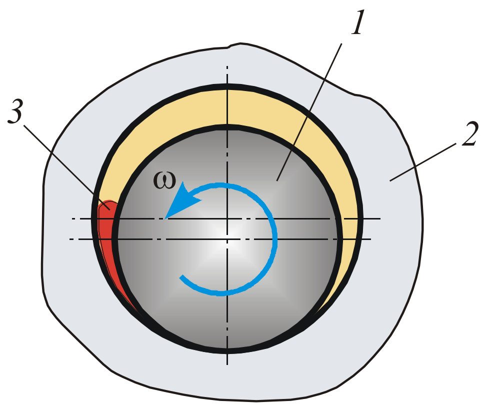


Fig. 11.23. Hydrodynamic lubricated bearing

Define the coefficient of fluid friction for bearing.

Newton experimentally derived the formula for determination of a force needed for shearing of one fluid surface in parallel with another. This force is called viscous force:

$$F = \mu S \frac{dV}{dh}.$$

Here $\frac{dV}{dh}$ – velocity gradient (characterizes velocity change throughout the fluid film thickness (Fig. 11.24)); S – shear area; μ – dynamic viscosity.

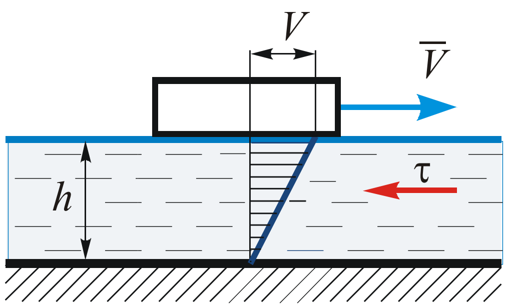


Fig. 11.24. Velocity gradient in the fluid film

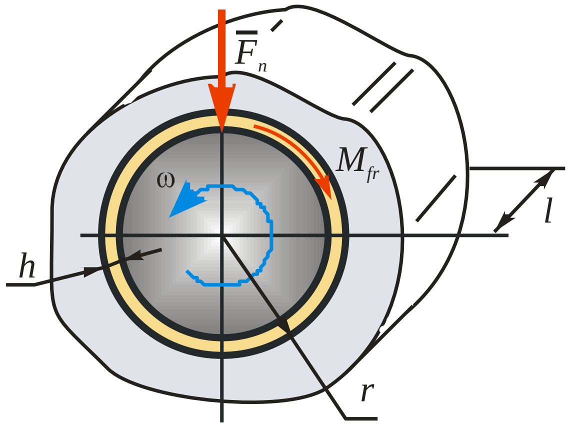


Fig. 11.25. Lubrication in the bearing

Viscous shear stress:

$$\tau = \frac{F}{S} = \mu \frac{dV}{dh},$$

and as dependence of velocity V on thickness h is linear (See Fig. 11.24), then:

$$\tau = \mu \frac{V}{h}.$$

Moment of friction in a bearing (See Fig. 11.25):

$$M_{fr} = F \cdot r = \mu \frac{V}{h} \cdot r \cdot S = \mu \frac{V}{h} 2\pi r^2 l = \frac{2\pi rn}{60} \frac{\mu}{h} 2\pi r^2 l = \frac{4\pi^2 r^3 nl}{60h} \mu. \qquad (11.10)$$

Here n – journal turning speed.

On the other side:

$$M_{mp} = F_n rf = 2\pi rlp \cdot rf \,. \qquad (11.11)$$

Here p – pressure in fluid.

Equating right sides of expressions (11.10) and (11.11), we will get the formula for determination of coefficient of fluid friction in a bearing:

$$f = \frac{\pi^2 r}{30h} \cdot \frac{\mu n}{p} \,.$$

The first multiplier in the expression characterizes geometric sizes, the second – friction regime. A diagram, which describes dependence of coefficient of fluid friction on friction regime, is shown in Fig. 11. 26. Marked zone shows the range, in which the most optimal friction conditions in a bearing are realized.

These expressions are true for lengthy journals. For short journals, as a result of oil leak through flanks, a friction pattern will be another. The pressure law in fluid film along the journal length for this case is shown in Fig. 11. 27.

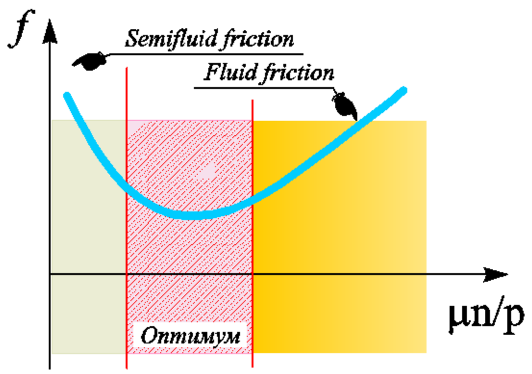


Fig. 11.26. Dependence of coefficient of fluid friction on friction regime

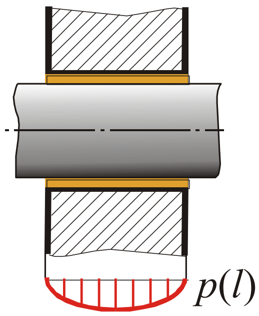


Fig. 11.27. The pressure diagram in fluid film

To perform the third and the fourth conditions it is necessary to secure such treatment of interacting surfaces to minimize surface roughness of the journal and bearing part, not to let significant journal strain, that can cause journal misalignments. It is also necessary to carry out qualitative filtration of oil.

11.4. WEAR-OUT OF ELEMENTS OF KINEMATIC PAIRS

In the process of exploitation of mechanisms and machines the wear-out of elements of kinematic pairs is observed. This is a harmful effect, because it decreases strength and toughness of details, lowers mechanism accuracy, distorts surface shape.

11.4.1. Wear modes

There are such main wear modes in mechanical engineering:

– mechanical wear as a result of mechanical effect;

– mechanochemical wear, when mechanical effect is accompanied by chemical or electrical interaction of material of rubbing bodies with medium;

– abrasive wear as a result of cutting or scratching of surface with particulate matter;

– erosion as a result of fluid or gas stream effect;

– fatigue wear connected with spalling of surface under repeated loading of surface layer (is typical for higher kinematic pairs);

– seizure wear as a result of seizure, tear of material and its transfer from one surface onto another (is typical for heavy specific pressure and sliding velocity in contact zone, for example, in screw-nut gear, worm gear, hypoid gear etc.)

Physical model of wear. During sliding in front of microasperity appears a roll of strained material (Fig. 11.28), in which compression stresses arise.

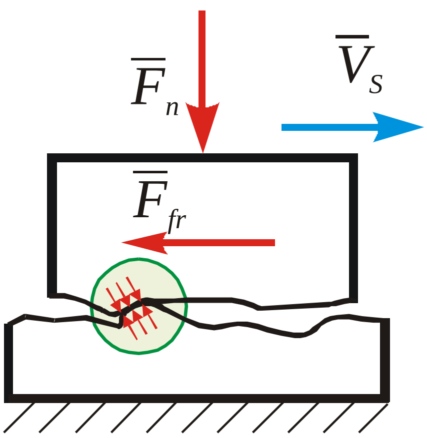


Fig. 11.28. Physical model of wear

Having passed the microasperity in this zone due to the friction the material stretches. That is, alternating stresses occur in the surface layer. We get fatigue of a material, in which are accumulated damages leading to separation of wear fragments.

11.4.2. Wear stages

There are two wear stages:

– first stage – attrition wear;

– second stage – normal wear or service wear.

At the attrition wear stage the initial surface roughness, obtained in the process of a detail production, changes, and before the second stage a new equilibrium roughness is formed, which doesn't change later on. At the stage of attrition the roughness can both decrease and increase. For example, a running-in process of polished detail is connected with increasing of its roughness to optimal.

To decrease the running-in time it is necessary to take tooling method, which could provide roughness, closest to equilibrium.

In Fig. 11.29 there is a graph of the change in the roughness parameter of the part – the arithmetic mean deviation of the profile R_a, mm – depending on the time of operation of the part.

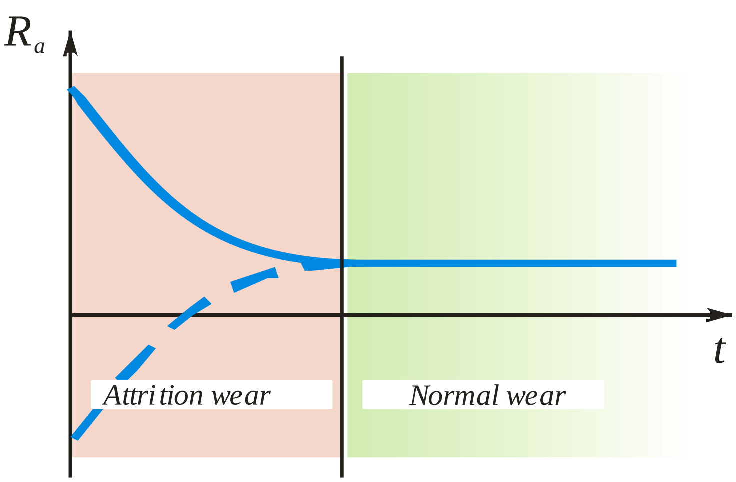


Fig. 11.29. Diagram of change of detail's roughness parameter

11.4.3. Quantitative assessment of wear-out

The result of wear-out is measured in units of length, volume or weight.

Limiting wear is wear that corresponds to the limit state of detail.

Permissible wear is wear, at which the operational integrity of machine part remains.

Graphic presentation of wear value distribution on the surface is called *wear distribution diagram.*

Wear rate:

$$\gamma = \frac{d\delta}{dt} = kp^m V_\kappa^n .$$

Here δ – thickness of destroyed layer of material; *p* – specific pressure in investigated point of a surface; V_κ – sliding velocity; *k* – wear factor (equals γ at $p = V_\kappa = 1)$; $m = 1\ldots3$ – coefficient, which depends on character of deformation in contact zone: elastic strain, plastic strain or there is micro-cutting; *n* – coefficient, which depends on wear mode.

For run-in pairs $m = n = 1$. Then:

$$\gamma = kpV_\kappa .$$

The expression pV_κ is called *friction power*.

Wear rate:

$$\gamma = \frac{d\delta}{ds}\frac{ds}{dt} = \gamma_s V_\kappa .$$

$\gamma_s = \dfrac{d\delta}{ds}$ – wear per unit of sliding distance, $\dfrac{\text{mm}}{\text{km}}$.

Wear resistance aka wearlessness is the ability of material to resist comprehensive external influences such as abrasion, cutting etc. during service.

The wear resistance of the parts is influenced by the hardness of the materials, their elastic properties, the mode of operation (sliding velocity V_κ, pressure on the contact area p, temperature t^o), external factors (lubrication, environment), design factors.

According to the value of γ_s there are three classes of materials:

– $\gamma_S = 10^{-12} \ldots 10^{-7} \dfrac{\text{mm}}{\text{km}}$ – material with high wear resistance at elastic strain;

– $\gamma_S = 10^{-7} \ldots 10^{-4} \dfrac{\text{mm}}{\text{km}}$ – material with a midrange of wear resistance at elastoplastic strain;

– $\gamma_S = 10^{-4} \ldots 10^{-3} \dfrac{\text{mm}}{\text{km}}$ – material with low wear resistance at micro-cutting.

11.5. FRICTIONAL POWER LOSSES. MECHANICAL EFFICIENCY

Energy, which is applied to the input link in a steady run spends to perform useful work and to overcome environmental resistance forces and frictions in kinematic pairs.

Efficiency is the ratio

$$\eta = \frac{A_{usf}}{A_{dr}}.$$

$A_{usf.}$ – work of useful resistance forces; A_{dr} – driving force work.

Efficiency can be expressed by corresponding powers averaged per cycle:

$$\eta = \frac{P_{output}}{P_{input}}.$$

The ratio of frictional power loss to the power input is called power-loss ratio:

$$\psi = \frac{P_{fr}}{P_{input}}.$$

Between efficiency and power-loss ratio there is such connection:

$$\psi = 1 - \eta.$$

If from the mechanism that is in a steady run, take off useful load (no-load conditions, when $P_{output} = 0$), than total power is

$$P_{input} = P_{output} + P_{fr} = P_{fr}.$$

It means that the power input is spent to overcome friction. At that $\eta = 0$, and $\psi = 1$.

Let's consider how to define the efficiency for different kinds of joining of mechanisms in machines:

11.5.1. Series coupling

The kinematic scheme of series coupling is shown in Fig. 11.30.

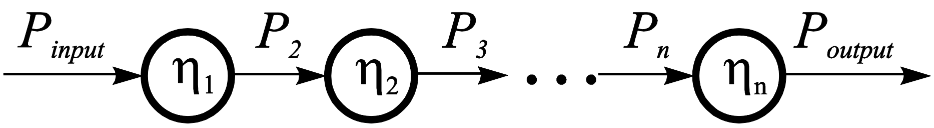


Fig. 11.30. Series coupling of mechanisms in machines

We suppose that the efficiency of mechanisms is known. Then we define the efficiency of machine. For the first mechanism $P_1 = P_{input}$ – power input for the whole machine. The power input for the second mechanism (according to the scheme it is power output of the first mechanism) $P_2 = P_1\eta_1$; for the 3rd – $P_3 = P_2\eta_2 = P_{input}\eta_1\eta_2$. For the *n*-th mechanism:

$$P_n = P_{input}\left(\eta_1 \cdot \eta_2 \cdot \ldots \cdot \eta_{n-1}\right).$$

Considering the *n*-th mechanism as the last in a chain, we find the power output of the whole machine as:

$$P_{output} = P_{input}\left(\eta_1 \cdot \eta_2 \cdot \ldots \cdot \eta_n\right).$$

The total efficiency

$$\eta = \frac{P_{output}}{P_{input}} = \eta_1 \cdot \eta_2 \cdot \ldots \cdot \eta_n .$$

11.5.2. Parallel coupling

The kinematic scheme of parallel coupling is shown in Fig. 11.31.

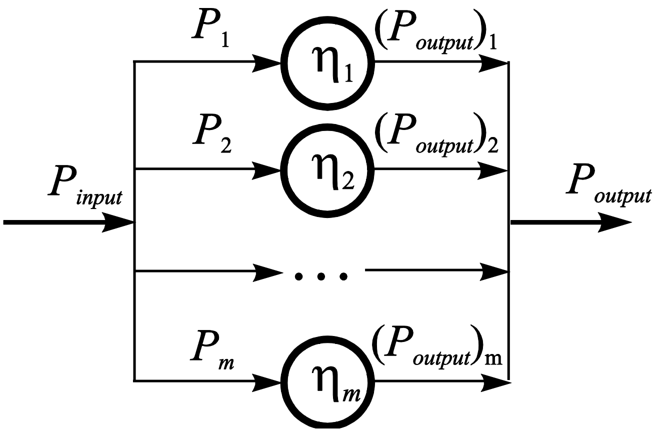


Fig. 11.31. Parallel coupling of mechanisms in machines

Here for each *i*-th mechanism power output is:

$$\left(P_{output}\right)_i = P_i \eta_i .$$

At the same time

$$P_{input} = \sum_{i=1}^{m} P_i ,$$

and

$$P_{output} = \sum_{i=1}^{m} \left(P_{output}\right)_i .$$

The efficiency of the whole machine is:

$$\eta = \frac{P_{output}}{P_{input}} = \frac{\sum_{i=1}^{m} \left(P_{output}\right)_i}{\sum_{i=1}^{m} P_i} = \frac{\sum_{i=1}^{m} P_i \eta_i}{\sum_{i=1}^{m} P_i}.$$

To define the efficiency in this case, it is necessary to know in which way the power input of the whole machine P_{input} is distributed between mechanisms. In other words it is necessary to introduce a coefficient known value:

$$\Omega_i = \frac{P_i}{P_{input}}.$$

Then

$$\eta = \frac{\sum_{i=1}^{m} \Omega_i P_{input} \eta_i}{P_{input}}.$$

Thus, total efficiency for such kinematic chains is calculated by the formula:

$$\eta = \sum_{i=1}^{n} \Omega_i \eta_i .$$

11.5.3. Series-multiple coupling

The kinematic scheme of series-multiple coupling is shown in Fig. 11.32.

Here for each parallel chain loop:

$$\left(P_{output}\right)_i = P_i \cdot \eta_{\Sigma i}, \quad (i = 1 \ldots m).$$

Here $\eta_{\Sigma i} = \eta_{i1} \cdot \eta_{i2} \cdot \ldots \cdot \eta_{im}$.

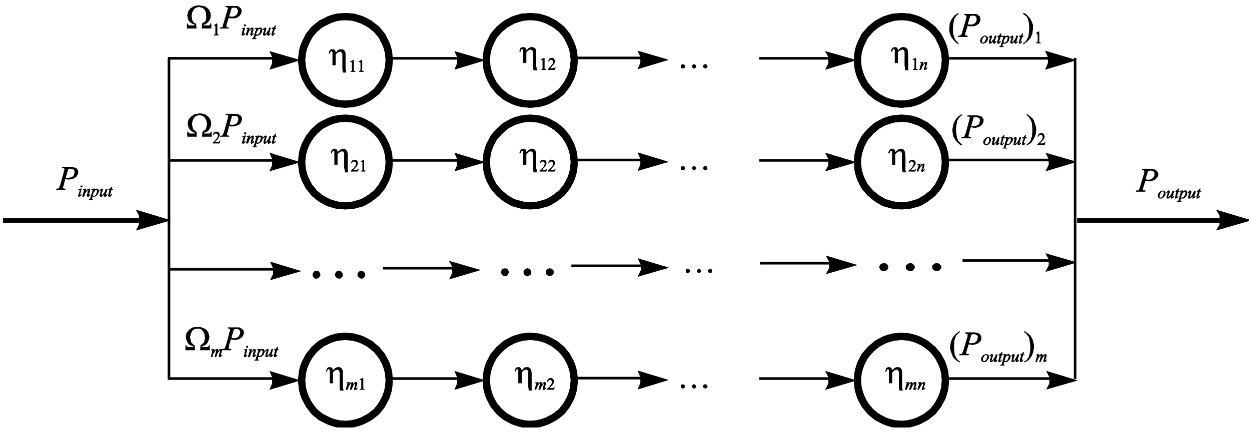


Fig. 11.32. Series-multiple coupling of mechanisms in machines

For the whole machine:

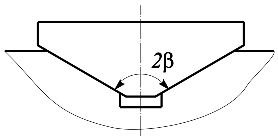
$$\eta = \frac{\sum_{i=1}^{m}\left(P_{output}\right)_i}{P_{input}} = \frac{\sum_{i=1}^{m} P_i \eta_{\Sigma i}}{P_{input}} = \frac{\sum_{i=1}^{m} \Omega_i P_{input}\left(\eta_{i1}\cdot\eta_{i2}\cdot\ldots\cdot\eta_{in}\right)}{P_{input}}.$$

Thus, efficiency for the whole machine is:

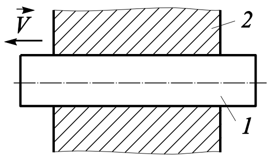
$$\eta = \sum_{i=1}^{m} \Omega_i \cdot \left(\eta_{i1}\cdot\eta_{i2}\cdot\ldots\cdot\eta_{in}\right).$$

QUESTIONS FOR SELF-TESTING

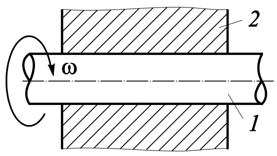
1. What is internal friction? Give examples of internal friction manifestations from the point of view of the mechanics of solids.
2. What is external friction?
3. What are the main types of friction that occurs in kinematic pairs?
4. Which friction force is greater: that which occurs during static or dynamic friction?
5. Formulate the law of dry friction of Amanton - Coulomb.
6. List the factors that influence the magnitude of the coefficient of friction.

7. 

Non-lubricated wedge-shaped slider made of bronze moves along the steel guide. Find the superficial coefficient of friction in the sliding paire if the angle $\beta = 60°$. The indicative values of the friction coefficients of different material pairs are given in Annex 8.

8. 

The cylindrical rod forms a translational pair with a guide. Find the superficial coefficient of friction if the pair is unruned. Rod and guide material is steel.

9. 

Find the superficial coefficient of friction in the turning run-in paire. The material of the links that make up this pair is steel.

10. Write down the equilibrium condition of forces applied to the nut at its screwdriving.
11. Formulate a condition for self-braking in a thread.
12. What should be the maximum tension force of the driven side of belt of the leather belt so that it does not slip relative to the steel belt pulley if the tension force in the leading side is 500 N? Spanning angle $\alpha = 120°$.
13. What is liquid friction?
14. What are the main kinds of lubricants?
15. What is the difference between hydrodynamic and hydrostatic lubrication?
16. List the characteristics of known you lubricants.
17. Formulate the conditions for liquid friction laid by N.P. Petrov as the basis of the theory of hydrodynamic lubrication.

18. What do we call a viscous force?
19. What is hydrodynamic lubricated bearing?
20. List the main types of wear in kinematic pairs.
21. What are the stages of wear.
22. In what units is the amount of wear and wear rate measured?
23. What wear is called permissible wear?
24. What is a limiting wear?
25. What is meant by wear resistance?
26. What factors affect the wear resistance of a part?
27. What is called the mechanical efficiency?
28. What is called the power-loss ratio and how is it related to the efficiency factor?
29. Can the power-loss ratio be equal to one. If so, under what conditions?
30. How to find the total efficiency of a kinematic chain with a series, parallel and series-multiple connection of its elements?

Chapter 12. VIBRATION IN MECHANISMS. VIBRATION PROTECTION

In modern high-dynamic machines there is mechanical oscillation. The source of this oscillation (vibrations) is mass unbalances of movable links (the first group of causes) and friction in kinematic pairs (the second group of causes).

If vibration is not a component of any process, so it will always be a hazard.

Lowering the vibration rate – *vibration protection* – is carried out in such directions.

Lowering the source vibration activity. The reduction of dynamic reacting forces by means of balancing of movable masses for the first group of causes and the utilization of special lubrication for the second group of causes are carried out.

Restyling of the unit. In such way we achieve:

– changing of natural frequency of construction units, which are depended on their geometry, i.e. elimination of resonance effects;

– increase of mechanical energy dissipation in the unit (*vibration damping*). It is achieved by means of choosing the materials of high absorptive or dissipative nature (wide elastic hysteresis loop); *structural damping* (friction in fixed joints – spline connections, threaded connections, riveted joints etc., where we have slight displacements in which work is carried out).

Dynamic antihunting. To the arrangement we add the *vibroextinguisher*, which generates oscillation that are in antiphase to those generated by the vibration source and, in such way, balances them.

Application of vibration absorbers – of *dampers*.

Vibration isolation. Its action results is in slackening of constraints between a source and an object. But some negative effects appear, such as additional objectionable unit displacements.

12.1. VIBRATION DAMPING

Fig. 12.1 and Fig. 12.2 show schemes of vibration suppressors, which are widely used in measuring instruments. They should provide such work of a device that it could respond keenly to a small signal. But after the cessation of this signal the meter needle should fluently return to „zero”. There should not appear spontaneous meter needle oscillations at a certain scale mark, which corresponds to a measured signal.

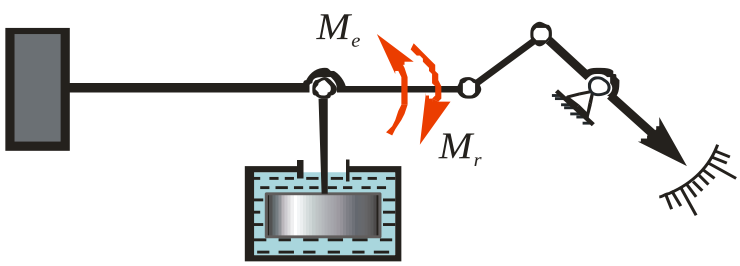


Fig. 12.1. Scheme of vibration suppressor I

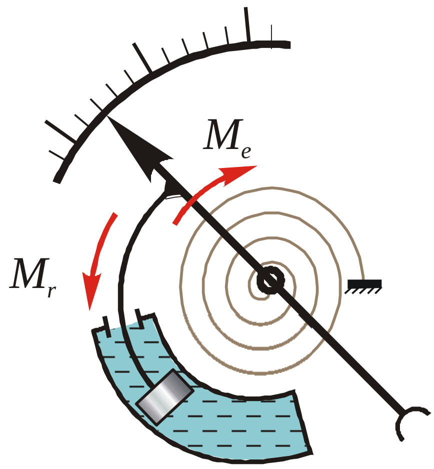


Fig. 12.2. Scheme of vibration suppressor II

Here M_e is the excitation moment; M_r is the moment of resistance forces.

Let’s study the example of analysis of a damper with rotating masses.

Resistance of damping medium is assumed as proportional to velocity (for liquid or viscous dampers) or as proportional to velocity squared (for air dampers):

Let’s write down the motion equation of a unit with viscous damper:

$$J\frac{d^2\varphi}{dt^2} + c\frac{d\varphi}{dt} + k\varphi + M_r = M_e. \qquad (12.1)$$

Here J is an inertia moment of movable masses; $C\dfrac{d\varphi}{dt}$ – a resistance moment of a damper.

Differential equation (12.1) describes vibration in a system.

Let's study such motion conditions:

1. We consider undamped system; resistance forces M_r are also absent. After object (meter needle) deflection the excitation moment M_e was unloaded. Then the equation (12.1) looks like:

$$J\frac{d^2\varphi}{dt^2}+k\varphi=0.$$

We obtained an ordinary differential equation of free vibration for single-degree-of-freedom system. The solution of this equation is

$$\varphi=\varphi_0\cos\left(\sqrt{\frac{k}{J}}t\right), \tag{12.2}$$

where the parameter φ_0 is the angle φ at $t=0$; $\sqrt{\frac{k}{J}}=\omega_0$ – circular frequency. The oscillation period $T=\frac{2\pi}{\omega_0}$; frequency $f=\frac{1}{T}$. So we have simple harmonic oscillation with period T and amplitude φ_0 (See Fig. 12.3).

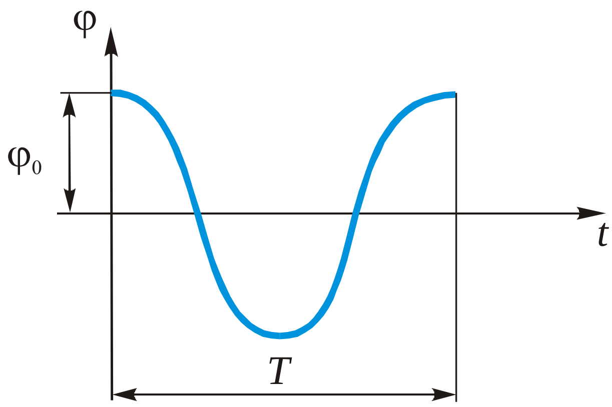


Fig. 12.3. Graff of harmonic oscillation

2. Let's introduce a damper. Then the equation (12.2) can be written down as:

$$J\frac{d^2\varphi}{dt^2}+c\frac{d\varphi}{dt}+k\varphi=0. \tag{12.3}$$

We mark

$$\beta=\frac{c}{2\sqrt{JK}}.$$

Here β is the damping factor.

$$\beta\omega_0=\frac{c}{2\sqrt{JK}}\omega_0 \quad \text{or} \quad \beta\omega_0=\frac{c}{2\sqrt{J}\cdot\sqrt{K}}\cdot\frac{\sqrt{K}}{\sqrt{J}}.$$

Hence

$$c=2\omega_0 J\beta.$$

We get

$$J\frac{d^2\varphi}{dt^2}+2\omega_0\beta J\frac{d\varphi}{dt}+k\varphi=0. \tag{12.4}$$

If $t=0$ $\varphi=\varphi_0$.

The solution of the equation (12.4) is:

$$\varphi=\varphi_0 e^{-\beta\omega_0 t}\left[\frac{\beta}{\sqrt{1-\beta^2}}\sin\left(\sqrt{1-\beta^2 t}\right)+\cos\left(\sqrt{1-\beta^2 t}\right)\right]. \tag{12.5}$$

Let's analyze the obtained expression. If $t\to\infty$, $\varphi\to 0$, i.e. damping takes place if $t\to\infty$ (Fig. 12.4), which is impossible.

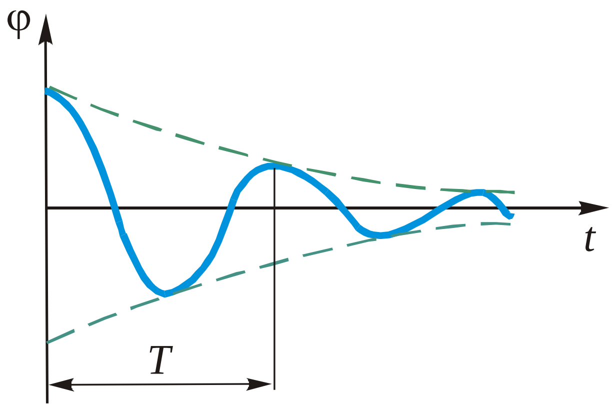


Fig. 12.4. Graff of the function (12.5)

Amplitude decay rate according to (12.5) is governed by the magnitude β (if $\beta = 0$ we get undamped oscillation (see equation (12.3)).

In practice they set some tolerance $\Delta\varphi$ (Fig. 12.5).

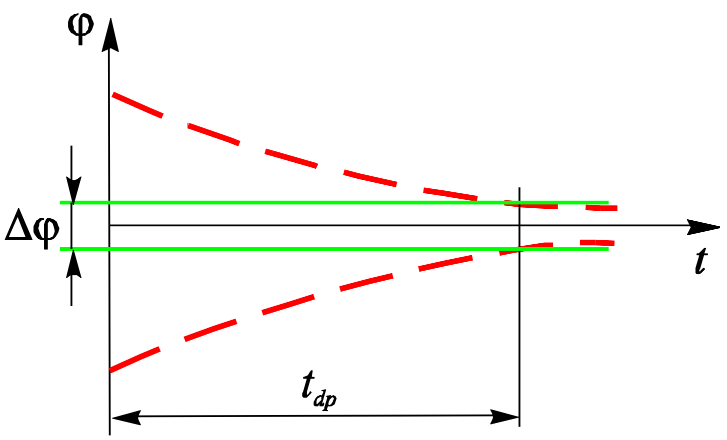


Fig. 8.5 Scheme of tolerance $\Delta\varphi$ determination

If parameters $\beta,\ J,\ k$ are set we can find t_{dp} – the damping time. In calculations, as a rule, the magnitude $\beta = 0{,}6\ldots 0{,}8$ is set.

So at damper analysis some magnitudes should be set (or defined):

$$\beta,\ J,\ c,\ \lambda,\ t_{dp}.$$

Here c is a spring constant, if it is present in a unit; $\lambda = \dfrac{\Delta\varphi}{\varphi_0}$ is a deflection tolerance.

One of the known formulas for defining of the damping time is the Arutunov's formula:

$$t_{dp} = \frac{1}{\beta\omega_0}\ln\frac{1}{\lambda\sqrt{1-\beta^2}} = \sqrt{\frac{J}{k\beta^2}}\ln\frac{1}{\lambda\sqrt{1-\beta^2}}.$$

or considering that $T = \dfrac{2\pi}{\omega_0}$, and $\sqrt{\dfrac{k}{J}} = \omega_0$,

$$\frac{t_{dp}}{T} = \frac{\ln\dfrac{1}{\lambda\sqrt{1-\beta^2}}}{2\pi\beta}. \qquad (12.6)$$

Fig. 12.6 shows a dependence diagram (12.6) for deflection tolerance $\lambda \approx 0{,}01$.

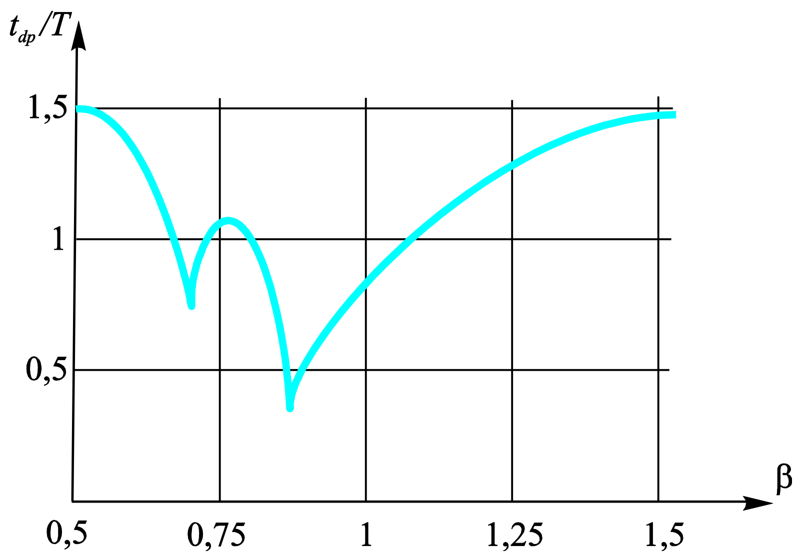


Fig. 12.6. Diagram of dependence (12.6) for deflection tolerance $\lambda \approx 0{,}01$

For spring-coupler damper we may use Rozumovsky's formula. According to this formula the spring constant is

$$c = \frac{2I}{t_{dp}} \ln \lambda .$$

12.2. STRUCTURES OF DAMPERS AND SHOCK ABSORBERS

12.2.1. Accelerative dampers

Rolling dampers (Fig. 12.7).

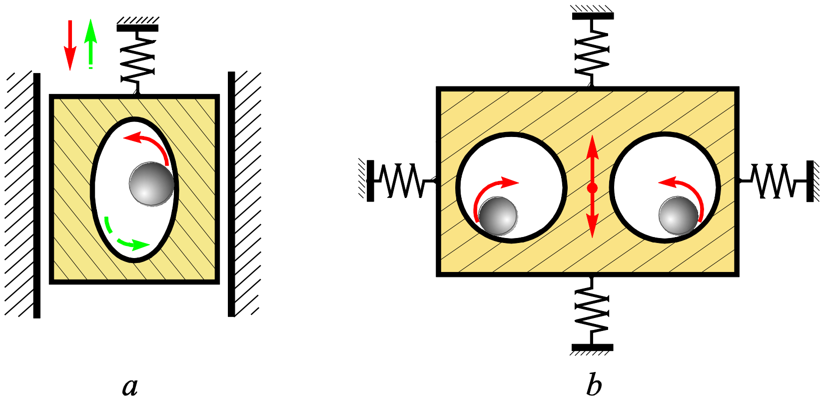


Fig. 12.7. Rolling dampers

The shape of the opening for the roller is important. So, if instead of a circle in the cross section, take an ellipse oblong in motion direction, then the role of higher harmonics in the spectrum of damper reactions increases. This is useful when the corresponding harmonics are present in the oscillations to be suppressed.

The design of the damper shown in Fig. 12.7, *b*, unlike the one shown in Fig. 12.7, *a*, allows us to do without guides, since in this case the lateral forces are mutually balanced.

Pendulum dampers (Fig. 12.8).

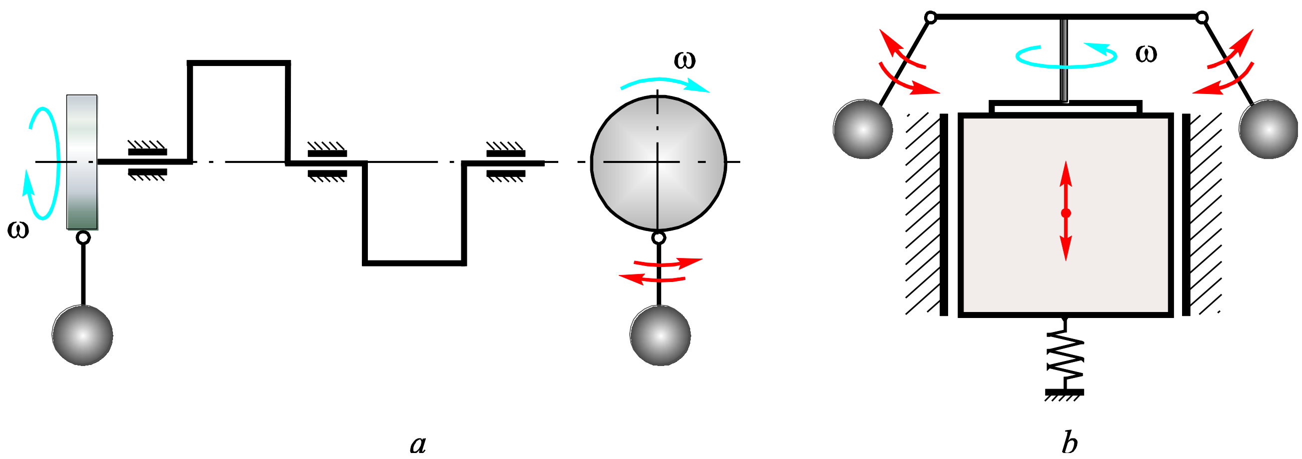


Fig. 12.8. Pendulum dampers: for suppression of torsional (*a*) and extensional (*b*) vibrations

Here are examples of dampers for suppression of torsional (Fig. 12.8, *a*) and extensional (Fig. 12.8, *b*) vibrations. Pendulums are under the action of centrifugal forces induced with movements of the object. Fluctuations in the speed of motion also cause fluctuations in the magnitudes and directions of the inertia forces, which, in fact, damp undesirable oscillating processes.

12.2.2. Air dampers

Dampers of such type are used in measuring instruments (Fig. 12.9 and Fig. 12.10).

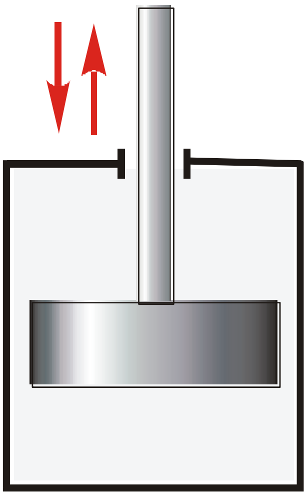


Fig. 12.9. Air damper I

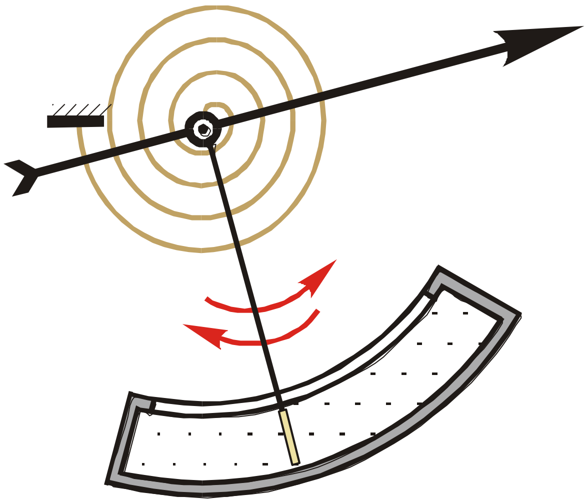


Fig. 12.10. Air damper II

12.2.3. Viscous dampers

Fig. 12.11 shows structure of a damper, which contains an adjusting bolt. With its help resistance to flow of liquid in pipe duct is changed and the damping time is controlled.

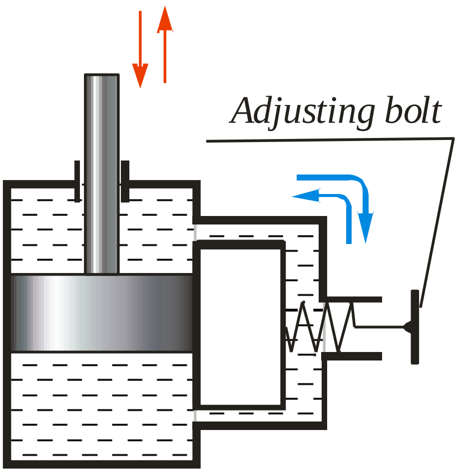


Fig. 12.11. Viscous damper I

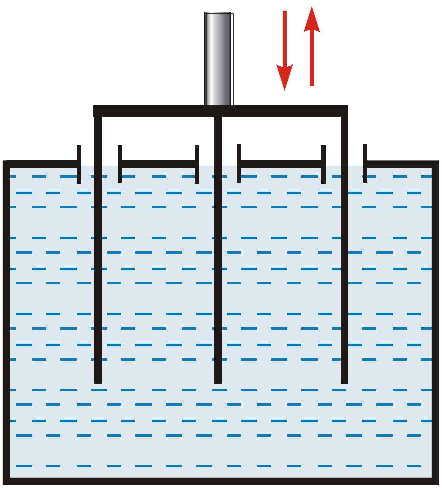


Fig. 12.12. Viscous damper II

In the unit, shown in the Fig. 12.12, the damping is held by increase of the resistance of liquid under immersion of plates (the greater the immersion depth, the bigger plate-liquid contact surface, so resistance force increases).

Fig. 12.13 shows a scheme of viscous damper for suppression of torsional vibrations.

As we see from the figure, when varying the blade geometry, we can control the resistance force to a wheel movement in liquid depending on rotational direction at vibrations. That is we can match necessary damper characteristics.

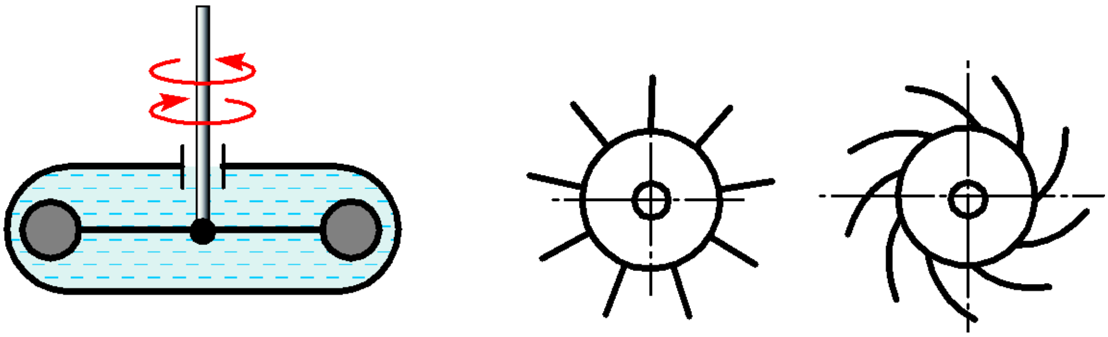


Fig. 12.13. Viscous dampers

Fig. 12.14 shows drip damper scheme used for movable objects damping in stiffly precision instruments, specifically in photoamplifier.

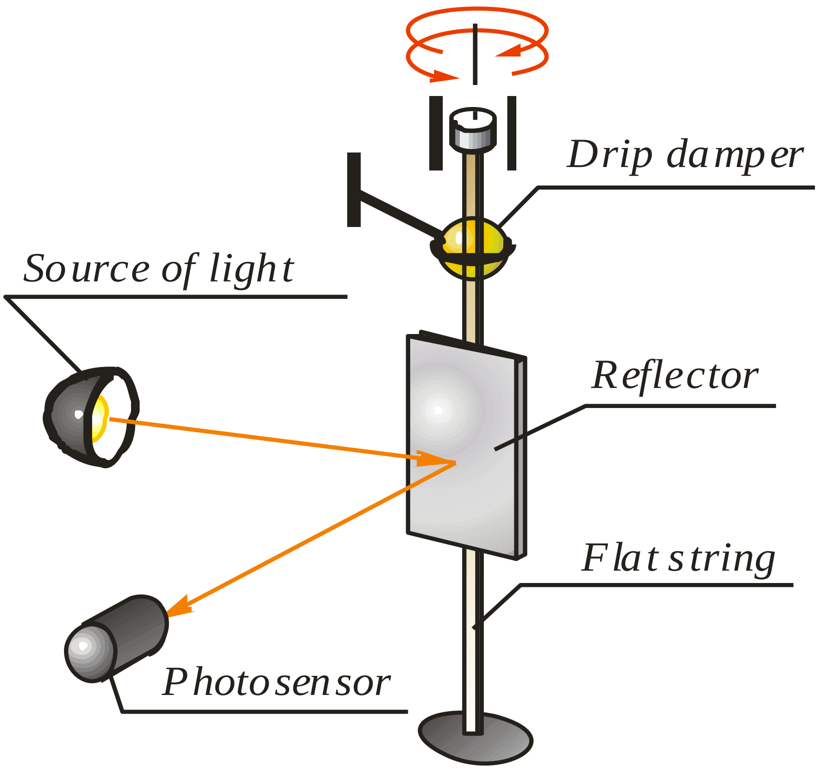


Fig. 12.14. Drip damper scheme

12.2.4. Shock absorbers

Unlike dampers, shock absorbers are used for high speed suppression. In Fig. 12.15 the scheme of the spring shock absorber is shown.

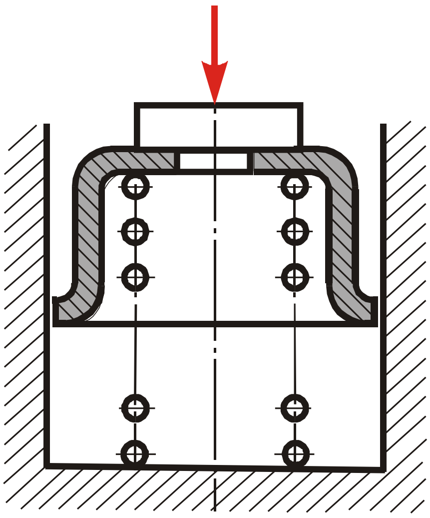


Fig. 12.15. Shock absorber

Shock absorbers can be also rubber, rubber-metal, rubber-spring, air-cushion and other types.

QUESTIONS FOR SELF-TESTING

1. What are the main sources of vibration in the mechanisms?
2. What are the main ways to reduce the negative effects of vibration on the operation of mechanisms and machines?
3. How to reduce the source vibration activity in modern machine aggregates?
4. How does redesign of the object affect the elimination of negative phenomena related to vibrations?
5. What does the term "vibration damping" mean? What are the main ways to implement it?
6. What is structural damping?
7. What is the essence of dynamic vibration damping?
8. What causes damping of vibrations in air and viscous dampers?
9. For what types of vibration suppressors do the resistances of their damping medium be proportional to the velocity of the object and for which to the squared velocity?
10. What are free oscillations?
11. Write down the equation of free oscillations of a mechanical system with one degree of freedom.
12. Write down the motion equations of an object with a viscous damper.
13. What principle is the work of inertial dampers based on?
14. What is the main purpose of shock absorbers? What is the principle of their work based on?
15. What are the main types of shock absorbers used in modern machine aggregates?

The values of the involute function $inv\alpha = tg\alpha - \alpha$

α°	Order of magnitude	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′
1	0,000	00117	00225	00281	00346	00420	00504	00598	00704	00821	00950	01092	01242
2	0,000	01418	01603	01804	02020	02253	02503	02771	03058	03364	03689	04035	04402
3	0,000	04790	05201	05634	06091	06573	07078	07610	08157	08751	09362	10000	10668
4	0,000	11364	12090	12847	13634	14453	15305	16189	17107	18059	19045	20067	21125
5	0,000	22220	23352	24552	25731	26978	28266	29594	30953	32394	33827	35324	36864
6	0,00	03845	04008	04175	04347	04524	04706	04892	05083	05280	05481	05687	05898
7	0,00	06115	06337	06564	06797	07035	07279	07528	07783	08044	08310	09582	08861
8	0,00	09145	09485	09732	10034	10343	10659	10980	11308	11643	11984	12332	12687
9	0,00	13048	13416	13792	14174	14563	14960	15363	15774	16193	16618	17051	17492
10	0,00	17941	18397	18860	19332	19812	20299	20795	21229	21810	22330	22859	23396
11	0,00	23941	24495	25057	25628	26208	26797	27394	28001	28016	29241	29875	30518
12	0,00	31171	31831	32504	33185	33875	34555	35285	36005	36735	37474	38224	38984
13	0,00	39754	40534	41325	42126	42938	43760	44593	45437	46291	47157	43033	48921
14	0,00	49819	50829	51650	52582	53526	54482	54448	56427	54717	58420	59434	60460
15	0,00	61488	62548	63611	64686	65773	66873	67985	69110	70248	71398	72561	73738
16	0,00	07493	07613	07735	07857	07982	08107	08234	08362	08492	08623	08756	08889
17	0,0	09025	09161	09299	09439	09580	09722	09866	10012	10158	10307	10456	10608
18	0,0	10760	10915	11071	11228	11387	11547	11709	11873	12038	12205	12373	12543
19	0,0	12715	12888	13063	13240	13418	13598	13779	13963	14148	14334	14523	14713
20	0,0	14904	15098	15293	15490	15689	15890	16092	16295	16502	16710	16920	17132
21	0,0	17345	17560	17777	17996	18217	18440	18665	18891	19120	19350	19583	19817
22	0,0	20054	20292	20533	20775	21019	21266	21514	21765	22018	22272	22529	22788
23	0,0	23044	23312	23577	23845	24114	24386	24660	24936	25214	25495	25778	26062
24	0,0	26350	26639	26931	27225	27521	27820	28121	28424	28729	29037	29348	29660
25	0,0	29975	30293	30613	30935	31260	31584	31917	32249	32583	32920	33260	33602
26	0,0	33947	34294	34644	34997	35352	35709	36069	36432	36798	37166	37537	37910
27	0,0	38287	38666	39047	39432	39819	40209	40602	40397	41395	41797	42201	42607
28	0,0	43017	43430	43845	44264	44685	45110	45537	45967	46400	46837	47276	47718

29	0,0	48164	48512	49064	49518	49976	50437	50901	51363	51838	52312	52788	52368
30	0,0	53751	54238	54728	55221	55717	56217	56720	57225	57736	58249	58765	59285
31	0,0	58809	60353	60856	61400	61937	62478	63022	63570	64122	64677	65236	65798
32	0,0	66364	66934	67507	68084	68665	69150	69838	70430	71026	71626	72230	72838
33	0,0	73449	74064	74684	75307	75934	76565	77200	77839	78483	79130	79781	80137
34	0,0	81097	81760	82428	83100	83777	84457	85142	85832	86525	87223	87925	88631
35	0,0	89342	90058	90777	91502	92230	92963	93701	94443	95190	95942	96698	97459
36	0	09822	09899	09977	10055	10133	10212	10292	10371	10452	10533	10614	10696
37	0	10778	10861	10944	11028	11113	11197	11283	11369	11455	11542	11630	11718
38	0	11806	11895	11985	12075	12165	12257	12348	12441	12534	12627	12721	12815
39	0	12911	13006	13102	13199	13297	13395	13493	13592	13692	13792	13893	13995
40	0	14096	14200	14303	14407	14511	14616	14722	14829	14936	15043	15152	15261
41	0	15370	15480	15591	15703	15815	15928	16041	16156	16270	16386	16502	16619
42	0	16737	16855	16974	17093	17214	17335	17457	17579	17702	17826	17951	18076
43	0	18202	18329	18537	18585	18714	18844	18975	19106	19238	19371	19505	19639
44	0	19774	19910	20047	20185	20323	20463	20603	20743	20885	21028	21171	21315
45	0	21460	21606	21753	21900	22049	22198	22348	22499	22651	21804	21958	23112
46	0	23268	23424	23582	23740	23899	24059	24220	24382	24545	24709	24874	25040
47	0	25206	25374	25543	25713	25883	26055	26228	26401	26576	26752	26919	27107
48	0	27285	27465	27646	27828	28012	28196	28381	28567	28755	28943	29133	29324
49	0	29516	29709	29903	30098	30295	30492	30691	30891	31092	31295	31498	31703
50	0	31909	32116	32324	32534	32745	32957	33171	33385	33601	33818	34037	34257
51	0	34578	34700	34924	35149	35376	35604	35833	36063	36295	36529	36763	36999
52	0	37237	37476	37716	37958	38202	38446	38693	38941	39190	39441	39693	39947
53	0	40202	40459	40717	40977	41239	41502	41767	42034	42302	42571	42843	43116
54	0	43390	43667	43945	44225	44506	44789	45047	45361	45650	43940	46232	46526
55	0	46822	47119	47419	47720	48023	48323	48635	48944	49255	49568	49882	50199
56	0	50518	50838	51161	51486	51813	52141	52472	52805	53141	53478	53817	54159
57	0	54503	54849	55197	55547	55900	56255	56612	56972	57333	57698	58064	58433
58	0	58804	59178	59554	59933	60314	60697	61083	61472	61863	62257	62653	63052
59	0	63454	63858	64265	64674	65086	65501	65913	66340	66763	67189	67618	68050

Appendix 2

Values of metric modules according to DSTU ISO 54-2001

Series	I	1 1,25 1,5 2 2,5 3 4 5 6 8 10 12 16 20 25 32 40 50
	II	1,125 1,375 1,75 2,25 2,75 3,5 4,5 5,5 (6,5) 7 9 11 14 18 22 28 36 45

Appendix 3

Recommended values of contact ratios

Tooth roughness parameter		Degree of accuracy				
		5	6	7	8	9
Ra *Rz*		0,63 –	1,25 –	2,5 –	– 20	– 40
ε_α	β=0	1,3	1,25-1,3	1,2-1,3	1,1-1,2	1,1-1,05
	β≠0	1,0	1,0	1,0	1,0	1,0

Appendix 4

Recommended limit values of tooth point thickness ratio depending on the structure of the material and the method of heat treatment

Non-hardened gears with homogeneous material structure	$s_a^* \geq 3$
Gears with teeth surface hardening	$s_a^* \geq 4$
Normalization, hardening and tempering	$s_a^* \geq 0,25...0,3$
Carburization, nitration	$s_a^* \geq 0,3...0,4$
Hardening	$s_a^* \geq 0,4...0,5$

Appendix 5

Values of worm modules

1-th series	0,10	-	0,125	-	0,16	0,20	0,25	-	0,315	0,40	0,50	-	0,63	0,80	-	1,0
2-th series	-	0,12	-	0,15	-	-	-	0,30	-	-	-	0,60	-	-	-	-
3-th series	-	-	-	-	-	-	-	-	-	-	-	-		-	0,90	-

Continuation of the appendix 5

1-th series	-	1,25	-	-	1,6	-	2,0	-	2,5	-	-	3,15	-	4,0	-	5,0	-
2-th series	-	-	-	1,5	-	-	-	-	-	-	3,0	-	3,5	-	-	-	6,0
3-th series	1,125	-	1,375	-	-	1,75	-	2,25	-	2,75	-	-	-	-	4,5	-	-

Continuation of the appendix 5

1-th series	6,3	-	8,0	-	10,0	-	-	12,5	-	16,0	-	20,0	-	25,0
2-th series	-	7,0	-	-	-	-	12,0	-	-	-	-	-	-	-
3-th series	-	-	-	9,0	-	11,0	-	-	14,0	-	18,0	-	22,0	-

Appendix 6

Worm-diameter factors

1-й ряд	6,3	-	8,0	-	10,0	-	12,5	-	16,0	-	20,0	-	25,0
2-й ряд	-	7,1	-	9,0	-	11,2	-	14,0	-	18,0	-	22,4	-

Appendix 7

Values of shift factor of basic rack for the following conditions: *a* – maximum increase in contact strength; *b* – bending strength; *c* – ensuring wear resistance and seizing resistance [11]

z_2	x	z_1														
		12			15			18			22			28		
		a	***b***	***c***	***a***	***b***	***c***	***a***	***b***	***c***	***a***	***b***	***c***	***a***	***b***	***c***
12	x_1	0,38	0,47	0,36												
	x_2	0,38	0,23	0,36												
15	x_1	0,30	0,53	0,43	0,45	0,58	0,44									
	x_2	0,50	0,22	0,34	0,45	0,28	0,44									
18	x_1	0,30	0,57	0,49	0,34	0,64	0,48	0,54	0,72	0,54						
	x_2	0,61	0,25	0,35	0,64	0,29	0,46	0,54	0,34	0,54						
22	x_1	0,30	0,62	0,53	0,38	0,73	0,55	0,60	0,81	0,60	0,68	0,95	0,67			
	x_2	0,66	0,28	0,38	0,75	0,32	0,54	0,64	0,38	0,63	0,68	0,39	0,67			
28	x_1	0,30	0,70	0,57	0,26	0,79	0,60	0,40	0,89	0,63	0,59	1,04	0,71	0,86	1,26	0,85
	x_2	0,88	0,26	0,48	1,04	0,35	0,63	1,02	0,38	0,72	0,94	0,40	0,81	0,86	0,42	0,85
34	x_1	0,30	0,76	0,60	0,13	0,83	0,63	0,30	0,93	0,67	0,48	1,08	0,74	0,80	1,30	0,86
	x_2	1,03	0,22	0,53	1,42	0,34	0,72	1,30	0,37	0,82	1,20	0,38	0,90	1,08	0,36	1,00
42	x_1	0,30	0,75	0,63	0,20	0,92	0,68	0,29	1,02	0,68	0,40	1,18	0,76	0,73	1,245	0,88
	x_2	1,30	0,21	0,67	1,53	0,32	0,88	1,48	0,36	0,94	1,48	0,38	1,03	1,33	0,31	1,12
50	x_1	0,30	0,58	0,63	0,25	0,97	0,66	0,32	1,05	0,70	0,43	1,22	0,76	0,64	1,22	0,91
	x_2	1,43	0,16	0,77	1,65	0,31	1,02	1,63	0,36	1,11	1,60	0,42	1,17	1,60	0,25	1,26

Appendix 8

Approximate coefficients of static and dynamic friction

Friction pair materials	Coefficient of friction			
	Static		Dynamic (sliding)	
	Dry friction	Lubricated friction	Dry friction	Lubricated friction
Steel – steel	0,15	0,1-0,12	0,15	0,05-0,1
Steel – soft steel	-	-	0,2	0,1-0,2
Steel – cast iron	0,3	-	0,18	0,05-0,15
Steel – bronze	0,12	0,08-0,12	0,10	0,07-0,10
Steel – textolite	-	-	-	0,02-0,06
Cast iron – bronze	-	-	0,15-0,2	0,07-0,15
Bronze – bronze	-	0,1	0,2	0,07-0,1
Rubber – cast iron	-	-	0,8	0,5
Metal – wood	0,5—0,6	0,1 - 0,2	0,3-0,6	0,1-0,2
Leather – metal	0,3-0,5	0,15	0,6	0,15

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ENGLISH-UKRAINIAN GLOSSARY

Термінологічний англо-український словник

A

Ability вміння.
Abrasion
1. (*син.* **attrition, deterioration, tearing, wear wear-out)** стирання, зношування, спрацювання.
2. Шліфування.

Abrasion resistance (*син.* **wearlessness, wear resistance)** опір зношуванню, зносостійкість.
Absence відсутність; недостатність.
Absolute shift абсолютне зміщення.
Absolute velocity абсолютна швидкість.
Absorptance поглинальна здатність.
Absorptive поглинальний, абсорбційний.
Abutment торець, п'ята, межа; опорний.
Acceleration прискорення
Accelerated motion прискорений рух.
Acceleration analogue аналог прискорення.
Acceleration diagram план прискорень.
Normal acceleration нормальне прискорення.
Tangential acceleration тангенціальне прискорення.
Total acceleration повне прискорення.
Accelerative damper інерційний гасник.
Accompany супроводжувати; проводити.
be accompanied (by) супроводжуватись.
Accrued зрослі (можливості), накопичений.
Accuracy точність.
Accuracy grade (*син.* **degree of accuracy, order of accuracy)** ступінь точності.
Accurate правильний, точний, калібрований.
Accurate toothing правильне зачеплення.
Achieve досягати, успішно виконувати; доводити до кінця.
Achievement досягнення.
Acid-resistant (*син.* **acid-fast, acid-proof)** кислотостійкий.
Acting діючий, робочий, працюючий; що виконує обов'язки.
Acting face робоча поверхня.
Acting profile of a camробочий профіль кулачка.
Action (*син.* **effect, impact, influence)** вплив.
Active активний, діючий, дійсний.
Active gear tooth flank (*син.* **working flank)** робочий профіль зубця
Acute angle гострій кут.
Addendum (*син.* **depth of pitch line)** висота головки зуба зубчастого колеса, головка зуба ЗК; коло вершин (*син.* **outside circle**).
Addition додаток, доповнення, приєднання, додавання (*мат.*).
Adherence зчеплення; щільне з'єднання, липучість (*для мастила*).
Adhesion [adhesive] power сила зчеплення.
Adjacent суміжний, прилеглий, сусідній (*to*).
Adjust ув'язувати.
Adjusting регулювальний; встановлювальний, складальний
Admissible допустимий.
Admission впуск, доступ, надходження; впускний.
Admission valve (*син.* **inlet valve, intake valve)** впускний клапан.
Admitted допустимий; визнаний, загальновизнаний.
Admitted region область допустимих значень.
Admitted range допустима область (діапазон).
Adsorption адсорбція, поверхневе вбирання.
Adsorption layer адсорбційний шар.
Advanced передовий, прогресивний.
Advantage перевага.
Adverse conditions несприятливі умови.
Algebraic алгебраїчний.
Alteration зміна, переробка.
Alternating змінний, перемінний; що чергується.
Ambit границі, діапазон, розмах, о́кіл (точки).
Amplitude амплітуда.
Amplitude decay убування за амплітудою.
Amplitude of oscillation амплітуда коливань.
Analytical method аналітичний метод.
Angular acceleration кутове прискорення.
Angle кут; кутовий; точка зору, погляд; косинець.
Angle of action (*син.* **pressure angle)** кут зачеплення; кут тиску.
Angle of friction кут тертя.
Angle of thread кут профілю різьби.
Cam angle of rotation кут повороту кулачка.
Involute angle евольвентний кут.
Lead angle кут підйому гвинтової лінії; кут підйому різьби.
Vertex angle кут при вершині.
Angular velocity кутова швидкість.
Angularly під кутом.
Annex (*син.* **appendix, addition)** додаток, доповнення.
Antifriction material антифрикційний матеріал.
Antihunting гасіння коливань.
Antiphase протифаза.
Apex верхівка, вершина.
Apex angle кут при вершині, кут розкриття (*конуса*).
Appear показуватися, з'являтися, здаватися; виявлятися; проявлятися.
Appendix (*син.* **annex)** доповнення, додаток (до книги)
Application практичне застосування.
Approach наближення, настання, підхід; наближатися, підходити.
Arbitrary довільний.
Arbitrary scale довільний масштаб.
Arbitrary shape довільна форма.
Arbitrarily chosen довільно вибраний.

Arc дуга.
Arc length довжина дуги.
Arc of circle (*син.* **circular arc)** дуга кола.
Arc of contact дуга зачеплення.
Archimedean worm архімедів черв'як.
Arithmetic mean середнє арифметичне.
Arm поводок.
Arm of force плече сили.
Arrangement монтаж, установлення; пристрій, механізм; розташування; класифікація; приведення до ладу; впорядкування.
As a rule як правило, зазвичай.
As few as possible (*син.* **a minimum of)** мінімальна кількість.
Aslant (*син.* **obliquely)** похило.
Aspect ratio співвідношення розмірів.
Asperities виступи шорсткості поверхні.
Asperity (*син.* **roughness)** шорсткість, шершавість; нерівність.
Assembling складання.
Assemblage of forces (*син.* **force system)** система сил.
Assertion твердження, формулювання (*мат.*).
Assign присвоювати, задавати.
Assigned profile заданий профіль.
Assur group група Ассура.
Assurance забезпечення, запевнення; гарантія.
As-turned finish (*син.* **surface roughness)** шорсткість поверхні.
At least хоча б, принаймні.
At right angle to … перпендикулярно до …
Attach (to) прив'язувати.
Attachment приєднання, з'єднання, прикріплення; приладдя, допоміжна деталь.
Attempt спроба; намагання; проба; пробувати, намагатися; братися.
Attribute ознака.
Attrition (*син.* **deterioration, tearing, wear, wear-out)** знос, зношування.
Attrition [attritious] wear знос під час приробляння.
Augmentation підвищення.
Availability (*син.* **working capacity, operational integrity)** роботоспроможність.
Average speed середня швидкість.
Axes осі.
Axial contact ratio (*син.* **overlap contact ratio)** коефіцієнт бічного перекриття.
Axial section осьовий переріз.
Axis вісь, осьова лінія; вал, шпиндель.
Axis of a journal вісь цапфи.
Axle base міжосьова відстань.
Axoid аксоїд.

B

Back-and-forth motion зворотно-поступальний рух.
Backlash зазор, коловий зазор в зубчастій передачі, люфт, мертвий хід.
Balanced state рівноважний стан.
Balancer балансир.
Balancing зрівноважування, балансування.
Ball-and-socket hinge сферичний шарнір.
Ball-to-ball contact контакт двох куль.
Barrel cam циліндричний кулачок.
Base фундамент, основа, базис; закладати основу.
Base circle основне коло.
Base of cone основа конуса.
Base pitch основний крок.
Base radiuses радіус основного кола, основний радіус.
Base tangent (*син.* **common normal)** спільна нормаль (*зубчасте зачеплення*).
Basic найважливіші факти, основи (чогось); основний.
Basic dimensions основні розміри.
Basic link базисна ланка.
Basic property основна властивість.
Basic rack зубчаста [інструментальна] рейка, гребінка.
Basic rack tooth profile вихідний твірний контур [ВТК] рейки.
Barren backoff непродуктивні втрати потужності.
… be accompanied (by) супроводжуватись.
Beam (*син.* **connecting rod, con-rod, coupler)** шатун.
Beam-and-crank mechanism кривошипно-шатунний механізм.
Bearing part (*син.* **bearing)** підшипник.
Become disconnected роз'єднуватись.
Below (*син.* **from below, underneath)** знизу.
Belt пас.
Belt pulley шків привідного паса.
Belt transmission пасова передача.
Bench стенд.
Bending strength міцність на згин.
Best-case value найкраще значення.
Beyond (*син.* **outside, out of)** поза, над, понад, вище.
Bevel скіс, загострення, фаска; косий, скісний.
Bevel gear конічна зубчаста передача; конічне зубчасте колесо
Bevel gear drive (*син.* **bevel gear)** конічна зубчаста передача.
Bevel gear wheel (*син.* **bevel gear, bevel wheel)** конічне зубчасте п колесо.
Bevel gearing конічне зубчасте зачеплення, конічна зубчаста передача.
Blade лезо, лопать.
Borderline lubrication (*син.* **thin-film lubrication)** граничне змащення.
Both обидва, той і другий.
Bottom land дно западини (*зубчастого колеса*).
Bound (*син.* **boundary)** границя, межа.
Break злам, отвір; тріщина, розколина, щілина, ламати(ся); розбивати(ся); рвати(ся); розривати(ся); руйнувати(ся); зламувати.
Break of a curve злам кривої.
Breast лінія вершин зубців.
Bulk properties об'ємні властивості.
By hand вручну.

C

Calibrated orifice калібрований отвір.
Cam кулачок.
  Barrel cam циліндричний кулачок.
  Cam angle of rotation кут повороту кулачка.
  Cam lobe контур кулачка.
  Cam mechanism (*син.* **cam box, cam gear**) кулачковий механізм.
  Conical cam конічний кулачок.
  Conjugate cam дводисковий кулачок.
  Conoid cam коноїдний кулачок.
  Diametrical cam діаметральний кулачок.
  Face cam (*син.* **track cam**) пазовий кулачок.
  Hyperboloid cam гіперболоїдний кулачок.
Cancel відміна, закреслювання, анулювати, відміняти, скорочувати (дріб, рівняння).
Cantilever bar консольний стержень; консольна балка.
Carrying capacity вантажопідйомність.
Cast iron чавун.
Cause причина.
Centre центр.
  Centre distance міжцентрова відстань.
  Centre distance increment factor (*син.* **coefficient of effective addendum modification**) коефіцієнт сприйманого зміщення.
  Centre line лінія центрів.
  Centre line of thrust лінія дії рівнодійної.
  Centre of curvature центр кривини.
  Centre of gravity центр ваги.
  Centre of mass центр мас.
  Centre of oscillation центр коливань (хитання).
  Centre of rotation центр обертання.
Centripetal acceleration доцентрове прискорення.
Centrode (*син.* **centroid line**) центроїда.
Certain деякий.
Cessation зупинка, призупинення, перерва.
Chain ланцюг.
  Chain loop гілка ланцюга.
Changing зміна, корекція параметрів.
Chord хорда.
Chord method метод хорд.
Chute спуск, жолоб, канал.
Circle коло, круг.
  Base circle основне коло.
  Pitch circle (*син.* **working pitch circle**) початкове коло.
  Standard pitch circle (*син.* **nominal pitch circle**) ділильне коло.
Circular коловий, круговий; круглий.
  Circular arc дуга кола.
  Circular notch width колова (ділильна) ширина западини зубчастого колеса.
  Circular pitch коловий крок.
  Circular pitch in plane of rotation (*син.* **transverse circular pitch**) торцевий крок.
  Circular tooth thickness колова (ділильна) товщина зубця зубчастого колеса.
Circumference довжина кола.
Circumscribe описувати.
Clarification ректифікація, очищення; перетворення, видозміна.
  Clarification filtration очистка фільтруванням.
Closing змикання, прилягання, закриття; запирання, закінчення, кінець.
Closure condition умова замкненості.
Clutch муфта.
Coefficient коефіцієнт.
  Coefficient of efficiency (*син.* **efficiency, coefficient of performance, COP**) коефіцієнт корисної дії.
  Coefficient of effective addendum modification (*син.* **centre distance increment factor**) коефіцієнт сприйманого зміщення.
  Coefficient of elasticity модуль пружності.
  Coefficient of fluid friction коефіцієнт рідинного тертя.
  Coefficient of friction (*син.* **friction coefficient**) коефіцієнт тертя.
  Coefficient of speed fluctuation коефіцієнт нерівномірності руху.
Cog-wheel зубчасте колесо.
Coincidence збіг, суміщення.
Combustion згорання.
  Combustion chamber камера згорання.
Combustible mixture робоча суміш (ДВС).
Common normal спільна нормаль.
Common tangent спільна дотична.
Compactness компактність.
Comparative estimation порівняльне оцінювання.
Comparatively відносно.
Compensate балансувати, зрівноважувати, компенсувати, надолужувати.
Compensating компенсація, компенсуючий.
Complement доповнювати; комплект, додаток.
Compliance згода, податливість, поступливість.
  in compliance with згідно з
Compound motion складний рух.
Concentrated force зосереджена сила.
Concurrence of lines перетин ліній.
Cone конус.
  Cone distance ділильна конусна відстань.
Condition of equilibrium умови рівноваги.
Confine обмежувати.
Conform узгоджуватися, підкорятися (правилам) пристосовувати(ся), зважати (на - to) ; погоджуватися (з - to).
Congruous відповідний.
Conical (*син.* **taper**) конічний.
  Conical cam конічний кулачок.
Conjugate з'єднаний, сполучений, спряжений (*мат.*).
  Conjugate cam дводисковий кулачок.
Connecting rod (*син.* **con-rod, coupler, beam**) шатун.
Conoid cam коноїдний кулачок.
Consecutive послідовний.
  Consecutive positions послідовні положення.
Constancy сталість.
Constant magnitude стала величина.

Continuous безперервний, суцільний, постійний (*про струм*).
Constituent складова, компонента.
Constraint в’язь, обмеження, напруженість, скутість.
Construction конструкція, будівництво, побудова.
Construction unit елемент конструкції.
Contact контакт, дотик, зв’язок.
Contact area площадка (поверхня) контакту.
Contact line лінія контакту.
Contact patch пляма контакту.
Contact point (*син.* **point of tangency, tangent point)** точка дотику.
Contact ratio коефіцієнт перекриття.
Contact stress контактне напруження.
Contact zone контактна зона.
Contacting bodies контактуючі тіла.
Contacting pair контактна пара.
Continuous безперервний, суцільний, постійний (*про струм*).
Continuous motion неперервний рух.
Contrary протилежний; зворотне, протилежне, протилежно, всупереч.
Contrary assertion протилежне твердження.
Converting перетворення.
Convex (*син.* **convex camber)** випуклість.
Co-ordinates система координат.
Coordinate origin початок координат.
Coordinated координований, узгоджений.
Coriolis acceleration прискорення Коріаліса.
Correction factor for profile shift (*син.* **shift factor)** коефіцієнт зміщення.
Corrective корегувальний.
Corresponding відповідний.
Corrosion корозія; роз'їдання; витравлювання; іржа.
Corrosion prevention захист від корозії.
Cosine косинус, косинусоїдальний.
Coulomb friction (*син* **dry friction**, **unlubricated friction)** сухе тертя.
Count рахувати.
Counteract зрівноважувати, протидіяти.
Crank (*син.* **crankshaft)** кривошип.
Crank-and-rod mechanism кривошипно-шатунний механізм.
Crank-and-slider mechanism кривошипно-повзунковий механізм.
Crank existence condition умова існування кривошипа.
Crank mechanism кривошипний механізм
Crank-rocker mechanism (*син.* **crank-guide, crank-and-slot mechanism, quick-return link mechanism,)** кривошипно-кулісний механізм.
Crankshaft колінчастий вал.
Cross хрест, перекреслювати, перетинати(ся), перехрещувати(ся), переходити.
Cross section (*син.* **crosscut)** поперечний переріз.
Crosscut поперечний переріз, поперечний.
Crosshead (*син.* **slider, sliding block, cylinder piston)** повзун.
Cumulative сумарний.
Curvature кривизна; вигин, згин; викривлення.
Curve крива (лінія); вигин; кривизна; закруглення; гнути, згинати; вигинати(ся).
Current поточний.
Current value поточне значення.
Current position поточне положення.
Smooth curve гладка крива.
Cutter різець; фреза; різак; різальний інструмент.
Shaping cutter довбач.
Cutting різання.
Cutting edge ріжуче лезо, вістря.
Cutting force сила різання.
Cutting machine металорізальний верстат.
Cutting motion рух різання.
Cycle цикл.
Cyclically циклічно.
Cycling циклічність.
Cyclogram циклограма.
Cycloid (*син.* **cyclic curve)** циклоїда.
Cycloidal циклоїдальний, циклоїдний.
Cycloidal gear циклоїдне зубчасте колесо, циклоїдна зубчаста передача.
Cycloidal gearing циклоїдальне зубчасте зачеплення.
Cylinder циліндр.
Cylinder piston (*син.* **crosshead, slider, sliding block)** повзун.
Cylinder-to-cylinder contact контакт двох циліндрів.
Cylindrical gear циліндричне зубчасте колесо, циліндрична зубчаста передача.
Cylindric(al) slider (*син.* **piston)** циліндричний повзун.
Cylindric(al) surface (*син.* **radial surface)** циліндрична поверхня.
Cylindrical worm (*син.* **single-enveloping worm)** циліндричний черв’як.

D

Damage пошкодження, пошкоджувати.
Damage accumulation накопичення пошкоджень.
Damp (*син.* **decay)** гасити, згасати, затухати, демпфірувати, гальмувати, амортизувати.
Damper демпфер, гасник, амортизатор.
Accelerative damper інерційний гасник.
Air damper повітряний гасник (демпфер).
Drip damper крапельний демпфер.
Liquid damper (*син.* **viscous damper)** рідинний гасник (демпфер).
Pendulum damper маятниковий гасник (демпфер).
Rolling damper котковий гасник.

Damped [decaying] oscillation затухаючі коливання.
Damping демпфування, затухання, гальмування.
Damping capacity демпфуюча здатність.
Damping characteristic характеристика демпфування.
Damping time час затухання (заспокоєння).
Structural damping конструкційне демпфірування.
Vibration damping демпфірування коливань.
Datum дана величина, елемент даних; вихідний рівень.
Datum point точка зведення.
Datum link ланка зведення.
Datums база, базова точка (лінія, площина), початок відліку; репер; точка (лінія, площина) зведення.
Dead weight власна вага.
Decay of oscillation затухання коливань.
Decelerate уповільнення; зменшувати швидкість (кількість обертів).
Decisive вирішальний, переконливий.
Dedendum висота ніжки зуба зубчастого колеса, ніжка зуба зубчастого колеса; коло западин (*син.* **root circle**).
Deepening заглиблення.
Defective layer дефектний шар.
Degradation деградація, виродження, погіршення.
Degree of accuracy (*син.* **accuracy grade, order of accuracy)** ступінь точності.
Degree of freedom ступінь свободи.
Delineate зображати, креслити, робити начерк.
Demand вимога; вимагати, потребувати.
Dependence залежність.
Depth глибина.
Derivative of function похідна функції.
Derivative sign знак похідної.
Design креслення, ескіз, рисунок; проектувати, конструювати, робити ескіз.
Design factor розрахунковий коефіцієнт.
Design formula (*син.* **design equation)** розрахункова формула.
Design size проектний розмір.
Design stage стадія проектування.
Deterioration (*син.* **attrition, tearing, wear, wear-out)** знос, зношування.
Develop розвивати(ся), конструювати, розробляти; розкладати (*мат.*); розгортати.
To develop on a plane розгортати на площину.
Development розвиток, зростання, розширення, розгортання; поліпшення, вдосконалення (*механізмів*); висновок, рішення; розгортка.
Device прилад, пристрій, пристосування; апарат.
Devise розробляти, винаходити, придумувати.
Diametrical cam діаметральний кулачок.
Differential диференціал (*мат., тех.*); диференціальний.
Differential gear (mechanism) диференціал (*тех.*), диференціальний зубчастий механізм.
Differentiation диференціювання.
Dilated domain розширена область.
Dimension розмір, вимірність.
Dimensions габарити.
Disadvantage недолік, невигідність, перешкода, завада.
Disengage роз’єднувати, виходити з зачеплення.
Disengagement роз’єднання, вихід із зачеплення.
Disk диск.
Disk-type gear cutter зуборізна дискова фреза.
Disparity нерівність, невідповідність.
Dissipation розсіювання (енергії), дисипація.
Dissipative розсіювальний.
Distinguish розрізняти, відрізняти(ся), відзначати(ся); побачити, помітити.
Dope (*син.* **grease, lubricating grease)** консистентне (густе) мастило.
Dotted line (*син.* **dot line, line dotted)** пунктирна лінія, пунктир.
Double подвійний, здвоєний, спарений; удвічі.
Double arm groups двоповідкова група.
Double-enveloping worm глобоїдний черв’як.
Double stroke подвійний хід (поршня).
Drag опір середовища.
Drip крапати; крапання.
Drip damper крапельний демпфер.
Drive передача, привід, привідний механізм.
Drive operation smoothness плавність роботи передачі.
Drive power потужність привода.
Drive shaft привідний (головний) вал.
Drive stage ступінь передачі (механічної).
Driven ведений.
Driving ведучий, привідний.
Driving force рушійна сила.
Drop падати, крапати, крапля.
Drop temperature температура краплепадіння.
Dry friction сухе тертя.
Durability (*син.* **reliability)** надійність.
Duration (*син.* **time)** тривалість.
Dyad діада.
Dynamic динамічний, активний, діючий.
Dynamic friction (*син.* **kinetic friction)** тертя руху.
Dynamic viscosity динамічна в'язкість.
Dynamics динаміка, рушійні сили.
Dynamotor двигун-генераторний агрегат.
Dwell angle кут стояння (*для кулачків*).
High dwell angle кут дальнього стояння.
Low dwell angle кут ближнього стояння.

E

Each other один по одному, один одного.
Eccentric ексцентрик.
Eccentricity ексцентриситет.
Economy in use економічний (про машину, обладнання).
Edge край (*чогось*), вістря, лезо, грань; точити; загострювати.
Effect дія, вплив, явище, результат; робити, чинити; виконувати, здійснювати.
Effective дійовий, ефективний, чинний, наявний; корисний (*тех.*).

Effective line of action (*син.* **active portion of line of action)** активна лінія зачеплення.

Effective addendum modification сприймане зміщення.

Efficiency (*син.* **coefficient of efficiency, coefficient of performance, COP)** коефіцієнт корисної дії.

Elastic пружний, гнучкий, гума (шнур).

Elastic force пружна сила, сила пружності.

Elastic foundation пружна основа.

Elastic hysteresis loop петля пружного гістерезису.

Elasticity пружність.

Elastohydrodynamic lubrication еластогідродинамічне змащення.

Eliminate усувати, виправляти.

Elimination усунення, виключення; ліквідація, знищення.

Ellipse еліпс.

Embodiment втілення, об'єднання; включення; злиття.

End-capping (*син.* **locking)** замикаючий.

End-mill type gear cutter зуборізна пальцева фреза.

Engage входити в контакт, входити в зачеплення.

Engagement зачеплення.

Engine двигун, мотор.

Engine drive shaft головний вал двигуна.

Engineered value розрахункова величина.

Engineering машинобудування, інженерія, технічний.

Engineering data технічні дані (характеристики), технічна документація.

Engineering decision технічне рішення.

Engineering factors технічні характеристики.

Engineering kinematics кінематика механізмів.

Engineer's system of units британська система з основними одиницями: фут, секунда, слаг.

Engler unit градус Енглера.

Enhance збільшувати, посилювати.

Envelope обвідна (лінія).

Envelope curve обвідна лінія.

Inside envelope (curve) (*син.* **inner envelope curve)** внутрішня обвідна.

Outer inner envelope (curve) зовнішня обвідна.

Environment середовище, довкілля.

Environmental resistance forces сили опору середовища.

Equality рівність.

Equation рівняння.

Equation of motion рівняння руху.

Equidistant рівновіддалений, еквідистантний.

Equilibrium баланс, рівновага.

Equilibrium equation рівняння рівноваги

Equilibrium roughness рівноважна шорсткість.

Equivalent mechanism замінний (еквівалентний) механізм.

Erosion ерозія; роз'їдання, витравлення, поступове руйнування.

Error похибка, помилка.

Estimate оцінювати.

Essential важливий, суттєвий, необхідний.

Evolvent (*син.* **involute)** евольвента.

Except за винятком, крім; виключати, заперечувати, відкидати.

Except for (*син.* **except)** крім, за винятком.

Excitation збудження.

Exploitation експлуатація.

Exact точний.

Excess (*син.* **surplus)** надлишок.

Exemplify наводити приклад.

Exert діяти (про силу).

External зовнішній.

External friction зовнішнє тертя.

External toothing зовнішнє зачеплення.

Extreme position of the mechanism крайнє положення механізму.

F

Face (*тех.*) поверхня, зріз, фаска; лице, лицьовий бік; грань; полірувати, обточувати.

Face cam (*син.* **track cam)** пазовий кулачок.

Face width ширина зубчастого вінця (*ЗК*).

Fail невдача, зазнати невдачі, виходити з ладу, ламатися.

Fall падати; спадати, знижуватися; осідати; падіння; занепад; зниження; схил.

Fall angle (*син.* **return angle)** кут повернення.

Farther віддалений, більш віддалений.

Fatigue утома (*матеріалу*).

Fatty жировий.

Feature ознака, властивість.

Feed підтримувати; подавати сировину; живлення, подача матеріалу.

Feed motion рух подачі.

Fidelity точність**,** правильність.

Fidelity of reproduction точність відтворення.

Field environment експлуатаційні умови.

Fillet surface перехідна поверхня (*зуба зубчастого колеса*).

Finish (*син.* **burnish, furbish, glaze, grind, polish, slick)** шліфування, полірування.

Finite size кінцева величина.

Fit придатний; відповідний, годитися, бути придатним, постачати.

Fitted curves спряжені криві (з'єднувані при побудові, наприклад, траєкторії).

Fitting складання.

Fixed нерухомий, закріплений, постійний, стаціонарний.

Fixed joint (*син.* **permanent connection)** нерухоме з'єднання.

Fixed link нерухома ланка.

Flank clearance бічний зазор.

Flash temperature (*син.* **flash point)** температура спалаху; точка займання.

Flat площина, плоска поверхня, плоский, пласкиий; рівний.

Flat-faced follower плоский тарілчастий штовхач.

Flat generating gear плоске твірне колесо.

Flat-topped generating gear плосковершинне твірне колесо.
Float плавати; спливати, триматися на поверхні.
Flow текти, литися, струменіти; течія, потік.
Fluctuation нестійкість, коливання, хитання, флуктуація; вагання.
Fluently плавно, гладко; вільно.
Fluid текуче середовище (*рідина або газ*); рідкий, текучий.
Fluid friction рідинне тертя.
Fluid film thickness товщина мастильного шару.
Fluid wedge (*син.* **lubricating oil wedge, physical wedge)** мастильний клин.
Follower штовхач (*кулачкового механізму*).
Flat-faced follower плоский тарілчастий штовхач.
Pointed follower загострений штовхач.
Roller follower штовхач, споряджений роликом.
Sliding follower (*син.* **pusher, translating follower)** поступально рухомий штовхач.
Spherical mushroom follower сферичний грибоподібний штовхач.
Swinging follower (*син.* **oscillating follower, rocker, rocker follower)** хитний (поворотний) штовхач.
Force сила, долати опір, змушувати.
Elastic force пружна сила, сила пружності.
Force closure силове замикання.
Force couple пара сил.
Force polygon многокутник сил.
Force system (*син.* **assemblage of forces)** система сил.
Foregoing попередній; вищезазначений.
Form форма, обрис, формувати, складати, утворювати.
Form closure геометричне замикання.
Form of loading закон навантаження.
Form surface фасонна (криволінійна) поверхня.
Frame (*син.* **fixed link, fixed frame, housing)** стояк, рама, корпус, каркас.
Forming process (*син.* **form-cutting method)** метод копіювання.
Free vibrations вільні (власні) коливання.
Friction тертя, сила тертя.
Dry friction (*син.* **Coulomb friction, unlubricated friction)** сухе тертя.
Dynamic friction (*син.* **kinetic friction)** тертя руху.
External friction зовнішнє тертя.
Fluid friction рідинне тертя.
Friction by vibratory displacements тертя за вібропереміщень.
Friction circle круг тертя.
Friction coefficient (*син.* **coefficient of friction)** коефіцієнт тертя.
Friction(al) disk фрикційній диск.
Friction(al) force (*син.* **friction)** сила тертя.
Friction gear фрикційна передача.
Friction pattern характер (картина) тертя.
Friction power потужність тертя.
Friction torque момент тертя.
Friction work робота сил тертя.
Internal friction внутрішнє тертя.
Kinetic friction (*син.* **sliding friction)** тертя ковзання.
Lubricated friction тертя зі змащенням.
Pivoting friction тертя вертіння.
Rolling friction тертя кочення.
Rolling friction with slippage тертя кочення з проковзуванням.
Sliding friction (*син.* **kinetic friction)** тертя ковзання.
Static friction (*син.* **stiction)** тертя спокою.
Unlubricated friction тертя без змащення, сухе тертя.
From the direction of … з боку.
Fuel delivery (*син.* **fuel feeding)** подача пального.
Full повний, наповнений, завершений; дуже, сильно, повністю.
Full angle повний кут (кут у 360°).
Full line суцільна лінія.
Full revolution per cycle повний оберт за цикл.
Full turn повний оберт.
Fume пара, випар, дим, кіптява; випаровуватися (*fume away*).
Functioning функціонування.

G

Gap зазор, люфт, проміжок, щілина, інтервал, пробіл, пропуск, велика розбіжність, розрив.
Gas газ, щось газоподібне; виділяти газ; наповнювати газом.
Gas lubricant газова мастило.
Gas-dynamic lubrication газодинамічне змащення.
Gas-static lubrication газостатичне змащення.
Gas stream потік (струмінь) газу.
Gear зубчаста передача, шестірня, привід, механізм, апарат; прилад; пристрій; зчіплювати(ся) (*про зубці коліс*).
Bevel gear конічне зубчасте колесо, конічна зубчаста передача.
Gear toothing зубчасте зачеплення.
Chevron gear (*син.* **herringbone gear)** шевронне зубчасте колесо, шевронна зубчаста передача.
Cylindrical gear циліндричне зубчасте колесо, циліндрична зубчаста передача.
Gear blank заготовка зубчастого колеса.
Gear cluster блок зубчастих коліс.
Gear cutting зубонарізування, нарізання зубчастих коліс.
Gear-cutting hob зуборізна черв’ячна фреза.
Gear-cutting tool зуборізний інструмент.
Gear face торець зубчастого колеса.
Gear hobber (*син.* **hobber)** зубофрезерний верстат.
Gear tip (*син.* **addendum, point of a tooth)** головка зуба зубчастого колеса.

Gear teeth spalling викрашування зубчастої передачі.
Helical gear (*син.* helical) косозубе колесо, косозуба передача.
Hypoid gear гіпоїдне зубчасте колесо, гіпоїдна зубчаста передача.
Involute gear евольвентне зубчасте колесо, евольвентна зубчаста передача.
Screw gear (*син.* spiral gear) гвинтове зубчасте колесо, гвинтова зубчаста передача.
Skew axes gear зубчаста передача з мимобіжними осями.
Spur gear прямозубе циліндричне колесо, прямозуба циліндрична передача.
Worm gear черв'ячна передача, черв'ячне колесо.

Gearing зчеплення; зубчаста передача, привід.
Bevel gearing конічне зубчасте зачеплення, конічна зубчаста передача.
Cycloidal gearing циклоїдальне зубчасте зачеплення.
Gearing quality indicators показники якості зачеплення.
Helical gearing косозуба передача.
Herringbone gearing шевронне зубчасте зачеплення, шевронна зубчаста передача.
Hypoid gearing гіпоїдне зубчасте зачеплення, гіпоїдна зубчаста передача.
Involute gearing евольвентне зубчасте зачеплення, евольвентна зубчаста передача.
Multiple gearing (*син.* compound gear train) багатоступінчаста зубчаста передача.
Right-angle bevel gearing ортогональна конічна передача.
Skew axes gearing зубчасте зачеплення з мимобіжними осями.
Single gearing (*син.* two-gear train) одноступінчаста зубчаста передача.
Spatial gearing просторове зубчасте зачеплення, просторова зубчаста передача.
Spur gearing (*син.* spur gear) прямозуба циліндрична передача.
Toothed gearing зубчаста передача.

General загальний, головний, поширений, загальноприйнятий,

General form загальний вид.

Generalized coordinate узагальнена координата.

Generally (*син.* in the general case/way) в загальному випадку.

Generating твірний; який утворює.
Generating motion рух обкочування (огинання).
Generating process метод обкочування (огинання).
Generating ray (*син.* generating line) твірна.
Generating surface твірна поверхня.

Generator генератор.

Geometric(al) геометричний(а).
Geometric(al) constraint геометрична в'язь.
Geometric(al) diagram (*син.* vector diagram) векторна діаграма.
Graphical plotting графічна побудова.

Geometrician геометр.

Geometry геометрія.

Get to know in detail детально ознайомитись.

Governor регулятор.

Grade (*син.* nature, property, quality) якість.

Gradient градієнт; ухил, схил.
Velocity gradient градієнт швидкостей.

Graphic діаграма, рисунок, креслення, графік; графічний, поданий як креслення або графік.

Graphical (*син.* graphic) графічний.

Graphical differentiation графічне диференціювання.

Graphical-analytical method графоаналітичний метод.

Graphite графіт.

Grease (*син.* lubricating grease, dope) консистентне (густе) мастило.

Greasing substance (*син.* lubricant) мастильний матеріал.

Groove паз, виїмка, жолобок.

Ground (*син.* fixed link, housing, rack) стояк, основа.

Guaranteed гарантований.

Guide спрямовуюча, напрямний пристрій, куліса; передаточний важіль; вести; керувати, скеровувати; направляти.

Guideline рекомендація, загальний курс, напрямок.

Gyrating mass обертальна маса.
Gyration обертання, обертальний рух.

H

Harmful шкідливий.

Hatch штрихова лінія, штриховка.

Hatching (*син.* hatch) штриховка.

Hazard шкідливий фактор.

Heat rejection відведення тепла.

Heel pivot п'ята.

Helix спіраль.

Helical спіральний, гвинтовий, гелікоїдальний, косозубе колесо.
Helical gear (*син.* helical) косозубе колесо.
Helical gearing косозуба передача.
Helical motion (*син.* screw motion) гвинтовий рух.
Helical screw гвинт.
Helical tooth гвинтовий зуб.

Helpful корисний.

Henceforth відтепер, віднині, надалі.

Herringbone gear шевронне зубчасте колесо, шевронна зубчаста передача.

Herringbone gearing шевронне зубчасте зачеплення.

Hertz formula формула Герца.

High найвища точка, максимум, високо, сильно, інтенсивно.
High dwell angle кут дальнього стояння.
High-quality високоякісний.
High speed shaft швидкісний вал.
High tech сучасна технологія; високотехнологічний.

Higher вищий.
Higher harmonic вища гармоніка.
Higher pair вища пара.
Higher pairing element елемент вищої пари.
Hinder перешкоджати, заважати, бути перешкодою.
Hinge (*син.* **hinge pivot)** шарнір.
Hinge axis вісь шарніра.
Hinged-lever mechanisms шарнірно-важільний механізм.
Hitherto досі, до цього часу.
Hobber (*син.* **gear hobber)** зубофрезерний верстат.
Hold тримати, держати, утримувати, володіти, мати; стримувати, спиняти, опора.
Housing (*син.* **ground, fixed link, frame)** корпус, станина, рама, стояк.
Hydraulic theory of lubrication гідродинамічна теорія змащення.
Hydrodynamic гідродинамічний.
Hydrodynamic lubrication гідродинамічне змащення.
Hydrostatic гідростатичний.
Hydrostatic lubrication гідродинамічне змащення.
Hyperbolic(al) gearing (*син.* **hyperboloid gear drive)** гіперболоїдна зубчаста передача.
Hyperboloid гіперболоїд.
Hyperboloid cam гіперболоїдний кулачок.
Hyperboloid gearing гіперболоїдна передача.
Hyperboloid of revolution (*син.* **hyperboloid of rotation)** гіперболоїд обертання.
Hyperfine надтонкий.
Hyperstatic system (*син.* **redundant system, statically indeterminate system)** статично невизначувана система.
Hypoid gear гіпоїдне зубчасте колесо, гіпоїдна зубчаста передача.
Hypoid gearing гіпоїдне зубчасте зачеплення, гіпоїдна зубчаста передача.
Hypotenuse гіпотенуза.
Hysteresis гістерезис.
Hysteresis loop петля гістерезису.

I

Idle mode (*син.* **idling, no-load conditions)** режим холостого ходу.
Idling холостий хід, робота на холостому ході.
Image зображення, образ, картина; зображати, змальовувати.
Imaginary уявний.
Imaginary generating gear уявне твірне колесо.
Imbalance (*син.* **unbalance)** неврівноваженість.
Immediately миттєво, негайно, невідкладно.
Immersion занурення.
Impact удар.
Impossible (*син.* **inadmissible; intolerable)** неприпустимий.
Improve вдосконалювати, покращувати.
In-line follower (follower on line of cam's axis) центральній штовхач.
In compliance with згідно з.
In inverse proportion обернено пропорційно.
In the general case (way) (*син.* **generally)** в загальному випадку.
In the line of вздовж, у напрямку.
In the range в межах.
In theory (*син.* **theoretically)** теоретично.
Inadmissible (*син.* **intolerable, impossible)** неприпустимий.
Incorrect (*син.* **irregular, mis-, violent, wrong)** неправильний.
Increment приріст.
Inertia інерція; сила інерції.
Inequality нерівність (*мат.*), різниця в розмірі (кількості), несхожість, нерівність (поверхні).
Infinite нескінченний, безмежний
The infinite (*син.* **infinity)** нескінченність, безмежність, безмежний простір.
Infinitesimal displacement елементарне переміщення.
Infinity (*син.* **the infinite)** нескінченність, безмежність, безмежний простір.
Inhomogeneity неоднорідність.
Initial початковий, попередній.
Initial link початкова ланка.
Initial state вихідний стан.
Inject насильно вводити рідину, впорскувати; вдувати.
Inner module внутрішній ділильний модуль.
Input вхідні дані.
Input link вхідна ланка.
Insertion включення, введення.
Inside середина; внутрішній; всередині.
Inside envelope внутрішня обвідна (лінія).
Insoluble нерозчинний.
Install встановлювати, проводити, монтувати.
Instant axis (*син.* **instantaneous axis)** миттєва вісь обертання.
Instantaneous миттєвий, моментальний, одночасний.
Instantaneous axis (*син.* **instant axis)** миттєва вісь обертання.
Instantaneous centre of rotation миттєвий центр обертання.
Instantaneous [instant] screw axis миттєва гвинтова вісь.
Interaction взаємодія.
Interacting bodies взаємодіючі тіла.
Interference інтерференція.
Intermediate проміжний.
Internal внутрішній.
Internal-combustion engine двигун внутрішнього згорання.
Internal friction внутрішнє тертя.
Internal toothing внутрішнє зачеплення.
Interpenetration взаємопроникнення.
Interpret тлумачити, пояснювати; інтерпретувати.
Interrelation взаємозалежність, взаємозв'язок.
Intersect перетинатись.
Intersection лінія перетину.
Intolerable (*син.* **inadmissible, impossible)** неприпустимий.
Invent винаходити, створювати.

Invention винахід.
Inventor винахідник.
Inverse обернений; зворотний; перевернутий; протилежний.
  Inverse motion обернений рух.
  Inverse proportion обернена пропорція, обернена пропорційність.
Inversion інверсія, зміна порядку на зворотний (обернений).
  Inversion principle принцип інверсії.
  Kinematic inversion кінематична інверсія.
Inversely обернено; обернено пропорційно.
  Inversely oriented обернено орієнтований (направлений).
Involute евольвента, розгортка; спіральний, закручений; підносити до степеня.
  Involute angle евольвентний кут.
  Involute function евольвентна функція.
  Involute gear евольвентне зубчасте колесо, евольвентна зубчаста передача.
  Involute gearing евольвентне зубчасте зачеплення, евольвентна зубчаста передача.
  Involute worm евольвентний черв'як.
Irrational number ірраціональне число.
Irregular (*син.* **incorrect, mis-, violent, wrong)** неправильний.
Irregularity нерівномірність.
Isoline ізолінія.

J

Jack важіль; домкрат; піднімати домкратом (*jack up*).
  Jack lifting підйом домкратом.
Joint об'єднаний, спільний; точка сполучення, стик; з'єднувати, сполучати; припасовувати (частини).
  Joint solution сумісний розв'язок.
Journal шийка вала, цапфа.
  Journal misalignment перекіс підшипника ковзання.
Justify обґрунтувати.
Jamming заклинювання.

K

Keenly гостро, різко, сильно.
Kinematic кінематичний.
  Kinematic chain (*син.* **kinematics)** кінематичний ланцюг
  Kinematic diagram кінематична діаграма.
  Kinematic pair кінематична пара.
  Kinematic scheme кінематична схема.
  Kinematic viscosity кінематична в'язкість.
Kinematically similar кінематично подібний.
Kinematics кінематика.
Kinetic кінетичний.
  Kinetic energy кінетична енергія.
  Kinetic friction (*син.* **sliding friction)** тертя ковзання.
Kinetostatics кінетостатика.

L

Laid off звільнений.
Lateral бічний; горизонтальний; побічний; другорядний.
  Lateral force поперечна (бічна) сила.
Law закон.
  Law of motion закон руху.
Lay off відкладати (відрізок).
Layer шар, нашарування.
  Defective layer дефектний шар.
Lead angle кут підйому гвинтової лінії; кут підйому різьби.
Leave out виключати, не брати до уваги.
Leftmost position крайнє ліве положення.
Leg (*син.* **side)** сторона трикутника.
Lever важіль; рукоятка, плече важеля.
Leverage система важелів, важільний механізм.
Liable можливий, ймовірний.
Limit границя, межа; граничний розмір, допуск; інтервал значень; обмежувати.
  Limit(ing) point гранична точка.
  Limit state граничний стан.
  Limit(ing) value граничне значення.
Limitation обмеження.
Limiting обмежувальний, граничний, стримуючий.
  Limiting contour блокуючий контур.
  Limiting form гранична форма.
  Limiting wear граничний знос.
Line лінія, риска, штрих; спосіб дій; напрям.
  Line dotted (*син.* **dot line, dotted line)** пунктир, пунктирна лінія.
  Line of action лінія зачеплення, лінія дії (сили).
  Theoretical line of action (*син.* **pressure line)** теоретична лінія зачеплення.
  Effective line of action (*син.* **active portion of line of action)** активна лінія зачеплення.
Linear лінійний, витягнутий в лінію.
  Linear depentanizer лінійна залежність.
  Linear displacement лінійне переміщення.
  Linear function лінійна функція.
  Linear law лінійний закон.
  Linear velocity лінійна швидкість.
Link ланка, куліса; зв'язувати, з'єднувати, змикати (**together, to**), зчіпляти (тж. **link up**).
  Input link вхідна ланка.
  Output link вихідна ланка.
Linkage (*син.* **leverage, link mechanism**) важільний механізм.
Linking складання.
Liquid damper (*син.* **viscous damper)** рідинний гасник.
Load навантаження, вантаж.
Load-bearing (*син.* **load-carrying)** навантажений, той, що несе навантаження.
Load-deformation curve (*син.* **load-deflection curve)** крива залежності деформації від навантаження.

Loading навантажування, навантаженість.
Loading diagram (*син.* **design model)** розрахункова схема.
Loading condition характер навантаження.
Locus (*син.* **locus of points, point curve)** геометричне місце точок.
Longitudinal поздовжній.
Longitudinal force поздовжня (осьова) сила.
Lopsided нахилений, перекошений, односторонній, нерівномірний.
Lose contact (with) відриватись.
Loud гучний, гучно, голосно, сильно, дуже.
Loud noise гучний шум.
Loud noises гучні звуки.
Lower pair нижча пара.
Lowering зниження, зменшення.
Lubricant мастило, змазка.
Fluid lubricant (*син.* **lubricating liquid)** рідке мастило.
Gas lubricant газова мастило.
Semisolid lubricant пластичний мастильний матеріал.
Solid lubricant тверде мастило.
Lubricated змащений, масляний, слизький.
Lubricated friction тертя зі змащенням.
Lubricating змащування; змащувати; мастильний, змащувальний.
Lubricating fluid мастильна рідина.
Lubricating graphite змащувальний графіт.
Lubricating grease (*син.* **grease, dope)** консистентне (густе) мастило.
Lubricating liquid (*син.* **fluid lubricant)** рідке мастило.
Lubricating oil wedge (*син.* **fluid wedge, physical wedge** рідинний (масляний) клин.
Lubrication змащення.
Elastohydrodynamic lubrication еластогідродинамічне змащення.
Gas-dynamic lubrication газодинамічне змащення.
Gas-static lubrication газостатичне змащення.
Hydrodynamic lubrication гідродинамічне змащення.
Hydrostatic lubrication гідростатичне змащення.
Thin-film lubrication (*син.* **borderline lubrication)** граничне змащення.
Viscous lubrication змащення пластичним (густим) мастилом.
Lumped mass зосереджена маса.

M

Machine машина; піддавати механічній обробці; обробляти на верстаті.
Machine oil машинне мастило (олива, масло).
Main головний; головне, основне.
The main theorem of gearing основна теорема зачеплення.
Magnitude величина, розмір.
Magnitude of vector абсолютна величина вектора
Magnitude of vector magnitude модуль вектора.
Manufacturable (*син.* **practically feasible)** технологічний.
Manufacturing виробництво; обробка; промисловий.
Mass production масове, поточне, серійне виробництво.
Match добирати; підходити, відповідати; протиставляти; змагання.
Matched відповідний, узгоджений.
Mating спряження, з’єднання, зчленування; входження в зачеплення.
Mating pair спряжена пара, пара тертя, фрикційна пара.
Mating surfaces спряжені поверхні.
Mating profiles спряжені профілі.
Maximum
1. **(***син.* **maxima)** максимум; найвищий ступінь.
2. **(***син.* **maximal)** максимальний.
Maximum possible максимально можливий.
Measure міра, одиниця виміру, масштаб, мірило, критерій; дільник (*мат.*); міряти, вимірювати, відміряти; мати розміри.
Measurement визначення розміру, вимірювання.
Measuring instrument вимірювальний прилад, засіб вимірювань.
Mean level (*син.* **midrange)** середній рівень.
Mechanism механізм, апарат, конструкція, пристрій; техніка (виконання).
Actuating mechanism (*син.* **actuator)** виконавчий механізм.
Beam-and-crank mechanism кривошипно-шатунний механізм.
Cam mechanism (*син.* **cam box, cam gear)** кулачковий механізм.
Crank mechanism кривошипний механізм.
Crank-and-rod mechanism кривошипно-шатунний механізм.
Crank-and-slider mechanism кривошипно-повзунковий механізм.
Crank-rocker mechanism (*син.* **crank-guide, quick-return link mechanism, crank-and-slot mechanism)** кривошипно-кулісний механізм.
Differential mechanism (gear) диференціал (*техн.*), диференціальний механізм.
Equivalent mechanism замінний механізм.
Geared linkage mechanism зубчасто-важільний механізм.
Hinged-lever mechanisms шарнірно-важільний механізм.
Link mechanism (*син.* **leverage, linkage**) важільний механізм.
Planar mechanism плоский механізм
Planetary mechanism планетарний механізм.
Primary (elementary) mechanism початковий механізм.

Mechanical машинний; механічний; автоматичний; технічний; машинальний.
Mechanical efficiency механічний к.к.д.
Mechanical wear механічний знос.
Mechanochemical wear корозійно-механічний знос.
Medium середина; середнє число; середовище; засіб, спосіб, шлях; середній, проміжний.
Mentioned вказаний.
Mercury (*син.* **quicksilver)** ртуть.
Mesh зачіпляти(ся); зчіпляти(ся)/
Meter needle стрілка вимірювального приладу.
Method метод; спосіб, система, порядок.
Microasperity мікронерівність.
Microcutting мікрорізання.
Midrange (*син.* **mean level)** середній рівень.
Milling cutter фреза.
Minimal найменший
Minimal boundary мінімальна границя.
Minimization мінімізація.
Misalignment неспіввісність, відхилення від осі; неточне суміщення; зміщення, розорієнтація; розбіжність.
Journal misalignment перекіс підшипника ковзання.
Mismatch не збігатися.
Mobility рухливість, мобільність.
Mobility degree ступінь рухливості.
Mode спосіб.
Module (*син.* **metric module)** модуль зачеплення.
Inner module внутрішній ділильний модуль.
Middle module середній ділильний модуль.
Normal module нормальний модуль.
Transverse module торцевий модуль.
Outer module зовнішній ділильний модуль.
Modulus абсолютне значення, абсолютна величина.
Moment момент (сили), момент (проміжок часу), мить.
Moment about момент відносно.
Moment arm плече сили.
Moment of couple момент пари сил.
Motion рух, хід (машини), приводити в рух.
Inverse motion обернений рух.
Motion distance величина переміщення.
Motion equation рівняння руху.
Motion link спрямовуюча.
Motion path траєкторія руху.
Motion reversal оберненість руху.
Motion transfer angle кут передачі руху.
Motion transmission передача руху.
Transportation motion переносний рух.
Uniform motion рівномірний рух.
Moveable connection рухоме з'єднання.
Moveable link рухома ланка.
Moving line твірна (поверхні обертання).
Movement рух; переміщення, пересування.
Movement judder нерівномірність (переривчатість) руху.
Multiple багаторазовий; багатократний; численний; складний, складений; кратний (*мат.*), кратне число (*мат.*).
Multiple gearing (*син.* **compound gear train)** багатоступінчаста зубчаста передача.
Multiply збільшувати(ся); множити (*мат.*), кратний.
Multiply by помножити на.
Multithreaded багатозахідний (черв'як, гвинт).
Multitude безліч.
Mutual взаємний.
Mutually balanced взаємно зрівноважений.

N

Natural природний, натуральний; справжній; звичайний; нормальний.
Natural frequency власна частота.
Natural oscillation власні (вільні) коливання.
Nib кінчик, виступ, клин, вістря.
Nitrogen азот.
Noise шум, перешкоди.
No-load conditions (*син.* **idle mode, idling)** режим холостого ходу.
Non-perpendicularity неперпендикулярність.
Non-ruled surface нелінійчаста поверхня.
Non-uniform нерівномірний.
Non-uniformity нерівномірність, неоднорідність.
Non-uniformly нерівномірно.
Normal нормальний, звичайний; правильний; перпендикулярний; середній.
Normal acceleration нормальне прискорення.
Normal line (*син.* **normal, perpendicular)** нормаль.
Normal module нормальний модуль.
Normal wear (*син.* **service wear)** нормальний (експлуатаційний) знос.
Notch западина (*ЗК*), проріз, паз, виїмка.
Numbers arrays масив чисел.
Number of revolutions швидкість обертання.
Number of threads (*син.* **number of starts)** кількість заходів.
Number of worm starts кількість заходів черв'яка.
Nut гайка, муфта.

O

Object об'єкт, предмет, річ; мета; заперечувати, противитися, протестувати (*to, against*).
Objectionable небажаний; що викликає заперечення.
Obliquely (*син.* **aslant)** похило, косо, навскіс, набік; поперек.
Obliquity нахил.
Oblong довгастий, видовжений; довгастий предмет; довгаста (подовжена) фігура.
Occur траплятися; відбуватися.
Offset відгалуження; галузь; зміщений.
Offset follower нецентральній штовхач.
Offset position зміщене положення.
Oil leak витік масла (мастила).
On/to/from the right (of) справа.
One more ще один.
Opening отвір, щілина; відкривання.

Operate діяти, працювати, приводити в рух.
Operating операційний, робочий (про режим), поточний
Operating condition умови експлуатації.
Operating mode робочий режим.
Operating speed робоча швидкість, частота обертання (*ел.*).
Operation дія, робота, процес; експлуатація; розробка, управління, керування.
Operational операційний, робочий; експлуатаційний.
Operational integrity роботоздатність; експлуатаційна придатність.
Optimal оптимальний.
Optimal choice оптимальний вибір (добір).
Order of accuracy (*син.* **accuracy grade, degree of accuracy,)** ступінь точності.
Ordinate ордината.
Ordinary differential equation звичайне диференціальне рівняння.
Origin початок, походження, джерело.
Orthogonal (*син.* **rectangular)** ортогональний, прямокутний.
Oscillating follower (*син.* **swinging follower, rocker, rocker follower)** хитний (поворотний) штовхач.
Oscillation (*син.* **vibration)** вібрація, коливання.
Out of (*син.* **beyond, outside)** за, зовні, вище, поза.
Out-of-balance condition неврівноважений стан.
Out-of-tolerance неточний; поза допуском.
Out-of-tolerance cut неточна обробка (*деталі*).
Outer module зовнішній ділильний модуль.
Outermost найвіддаленіший.
Outermost position найвіддаленіше положення.
Output продуктивність.
Output link вихідна ланка.
Outside зовнішня сторона (частина, поверхня); зовнішній; сторонній, що знаходиться зовні; поза, за межами, за межі; крім, за винятком.
Outside circle (*син.* **addendum)** коло вершин.
Outside radius радіус кола вершин.
Overcome перемогти, побороти; подолати.
Overlap перекривати, заходити одне за одне; частково покривати; перекриття (*тех.*).
Overlap contact ratio (*син.* **axial contact ratio)** коефіцієнт бічного перекриття.
Oxide окисел, оксид.
Oxide film оксидна плівка.

P

Pair пара (два однакових предмети).
Pair of compasses циркуль.
Pair-wise interaction попарна взаємодія.
Pairing element елемент кінематичної пари.
Parallel паралельний.
Parallel coupling паралельне з'єднання.
Parallel motion поступальний рух.
Parallel motion link поступально рухома ланка.
Parallel-plane movement плоско-паралельний рух.
Part деталь, частина; частково (*син.* partly); відділятись, від'єднуватись.
Part restyling (*син.* **restyling)** зміна конструкції деталі.
Particle матеріальна точка.
Particulate matter тверді часточки, порошок.
Pass рухатися вперед; проходити; перетинати; переходити; перевищувати, виходити за межі; витримати, пройти (випробування) ; відповідати (вимогам); зникати; припинятися.
Pass on переходити.
Passing проходження; побіжний, випадковий.
Passive constraint пасивна в'язь.
Peculiarity особливість, специфічність, властивість, характерна риса.
Pendulum damper маятниковий гасник.
Penetration проникнення, проникливість.
Perform виконувати, здійснювати.
Performance виконання, здійснення; характеристика (*роботи машини тощо*) ; експлуатаційні якості; продуктивність; коефіцієнт корисної дії.
Permanent connection (*син.* **fixed joint)** нерухоме з'єднання.
Permissible (*син.* **admissible, possible)** дозволений, припустимий.
Permissible wear допустимий знос.
Permit дозволяти, давати дозвіл; надавати можливість; допускати.
Perpendicular planes взаємно перпендикулярні площини.
Persistence сталість, інерційність.
Phenomenon явище, феномен.
Physical model фізична модель.
Photoamplifier фотопідсилювач.
Piecewise кусочно-лінійний.
Pilot analysis пілотний (передній) аналіз.
Pin палець; цапфа; штифт, болт; вісь; шплінт; протинати; пробивати.
Pin gear цівкова зубчаста передача.
Pin joint шарнірне з'єднання, шарнір.
Pin wheel цівкове колесо.
Pinion шестерня.
Pipe труба; трубопровід; пускати трубами.
Pipe duct трубопровід.
Piston поршень.
Piston stroke хід поршня.
Pitch крок, ступінь, рівень, пітч, зачіплювати (*про зубці*).
Base pitch основний крок.
Circular pitch коловий крок.
Pitch angle кут початкового конуса (*ЗП*).
Pitch cone початковий (ділильний) конус.
Pitch curve теоретичний профіль (*кулачка*).
Pitch distance крок, величина кроку.
Pitch of thread крок різьби.
Pitch point полюс зачеплення.
Pivot шарнір, цапфа; точка опори; точка обертання; стержень; вісь; крутитися, обертатися; надівати на стержень.
Pivot(al) point точка [вісь] повороту.
Pivoting friction тертя вертіння.

Planar [plane] mechanism плоский механізм.
Plane площина; плоска поверхня; грань; проекція; рівень (розвитку, знань тощо); крило (літака); пласкиий, плоский; площинний; рівняти, вирівнювати.
Plane curve плоска крива.
Plane [planar] mechanism плоский механізм/
Plane of changing площина корекції.
Plane pinion сателіт.
Planetary gearing (*син.* **planetary gear mechanism)** планетарний зубчастий механізм.
Playground майданчик для гри; спортивний майданчик.
Point точка; момент (часу); крапка (в десяткових дробах); поділка шкали; мета, намір; вістря, гострий кінець; кінчик; наконечник.
Point curve (*син.* **locus, locus of points)** геометричне місце точок.
Point of attack точка прикладання.
Point of force application точка прикладання сили.
Point of inflection точка перегину.
Point of tangency (*син.* **contact point, tangent point)** точка дотику.
Point of a tooth (*син.* **gear tip, addendum)** головка зуба зубчастого колеса.
Pointed загострений, гострий.
Pointed follower загострений штовхач.
Pointing загострення, заточування; зазначення (*напрямку, місця тощо*).
Poise пуаз.
Polar coordinates полярні координати.
Pole полюс; полюсний.
Pole of acceleration diagram полюс плану прискорень.
Pole of inertia полюс інерції.
Pole of velocity diagram полюс плану швидкостей.
To be poles apart бути діаметрально протилежним.
Polished відполірований**.**
Position положення; звичайне (правильне) місце; ставити; розташовувати.
Leftmost position крайнє ліве положення.
Position function of a mechanism функція положення механізму.
Position of extremum точка екстремуму.
Position of load application точка прикладання навантаження.
Position vector (*син.* **radius-vector)** радіус-вектор.
Rightmost position крайнє праве положення.
Positional relationship взаємне розташування.
Positive позитивний, реальний, точний; додатний (*мат.*); примусовий (про рух).
Positive closing примусове замикання.
Powder-like порошкоподібний, пиловидний.
Power сила; потужність, енергія; продуктивність; здатність, можливість; степінь (*мат.*).
Power train силова передача.
Power transmission механічна передача.
Power waste (*син.* **power loss)** втрати енергії.
Power input потужність, що підводиться, споживана потужність.
Power output вихідна потужність.
Power-loss ratio коефіцієнт утрат.
Practically feasible (*син.* **manufacturabl)** технологічний.
Precision точність, чіткість, акуратність; точний.
Precision instrument точний прилад.
Predecessor попередник.
Prescribed заданий (закон).
Pressure тиск; стискання; пресування.
Pressure distribution розподіл тиску
Pressure angle (*син.* **angle of action)** кут зачеплення; кут тиску.
Specific pressure (*син.* **unit pressure)** питомий тиск.
Prevailing (*син.* **outstanding, predominant, prevalent)** панівний, превалюючий, домінуючий, переважний, **(***син.* **widespread)** широко розповсюджений.
Prevalent (*син.* **abundant, ample, common, copious, plentiful, prevailing, rife, widespread)** розповсюджений, поширений, переважний.
Prevent попереджати, відвертати; запобігати, заважати, перешкоджати (чомусь - *from*).
Primary [elementary] mechanism початковий механізм.
Prime radius of a cam мінімальний радіус кулачка.
Primitive примітивний.
Principal головний, основний.
Principle принцип, правило, закон; першопричина; причина, джерело.
Procedure процедура, методика.
Process технологічний процес.
Processing оброблляння; обробка даних (*комп.*).
Processing equipment технологічне обладнання.
Product продукція, продукт, виріб; добуток (*мат.*); результат, наслідок.
Production продуктивність; виробництво; виготовлення, видобуток; продукція, виріб; виробничий.
Profile
1. Профіль, обрис, контур; вертикальний розріз, перетин; фасонний.
2. **(***син.* **shape)** профілювати.
Proportion співвідношення, пропорція.
Proportional пропорційний.
Pulley шків, блок.
Belt pulley шків привідного паса.
Push натискати; тиск, натискання.
Pusher (*син.* **translating follower, sliding follower)** поступально рухомий штовхач.

Q

Quadrangle чотирикутник.
Qualitative якісний.
Qualitative properties характеристики якості (*виробу*).

Quality (*син.* nature, property, grade) якість; властивість; особливість; характерна риса.

Quality coefficient (*син.* quality indicator) показник якості.

Quality metrics показники якості.

Quality rating оцінка якості.

Quantitative assessment кількісна оцінка.

Quantity кількість; величина (*мат.*).

Quantity of rotations per second кількість обертів за секунду.

Quick швидкий; швидко, скоро.

Quickly швидко.

R

Rack

1. **(*син.* fixed link, ground)** стояк (у структурній схемі).
2. **(*син.* basic rack)** зубчаста [інструментальна] рейка, гребінка.

Rack-shaped cutter (*син.* rack-type cutter) зуборізна рейка (гребінка).

Radial clearance (*син.* top clearance) радіальний зазор.

Radius-vector (*син.* position vector) радіус-вектор.

Radius of curvature радіус кривини.

Random довільний.

Range інтервал; класифікувати; коливатися в певних межах.

Rapprochement зближення.

Rare gas інертний газ.

Rarely рідко, нечасто; надзвичайно, винятково.

Rate відповідна частина; пропорція; коефіцієнт; ступінь; відсоток; частка; визначати; встановлювати.

Rate of rise ступінь (швидкість, крутизна) підйому.

Ratio відношення, пропорція; коефіцієнт; співвідношення; передатне число.

Ratio of side dimensions співвідношення розмірів сторін.

Rational number раціональне число.

Ray промінь, напівпряма.

Reaction (*син.* reacting force) реакція, сила протидії.

Reasonable раціональний, коректний, прийнятний.

Reciprocal sliding (*син.* reciprocal motion) зворотно-поступальне ковзання (рух).

Reciprocating motor поршневий двигун.

Rectangle прямокутник.

Rectangular (*син.* orthogonal) ортогональний, прямокутний.

Rectilinear (*син.* rectilineal) прямолінійний.

Rectilinear generator прямолінійна твірна.

Reduce зводити, зменшувати, скорочувати; приводити до спільного знаменника (*мат.*).

Reduced зведений, зменшений, скорочений.

Reduced coefficient of elasticity зведений модуль пружності.

Reduced inertia moment зведений момент інерції.

Reduced [equivalent] mass зведена маса.

Reduced forces зведена сила.

Reduction зведення, зменшення, зниження; скорочення; перетворення; приводення до спільного знаменника (*мат.*).

Reduction of masses зведення мас.

Reduction of forces зведення сил.

Reduction ratio (*син.* transmission ratio) передатне число.

Redundant зайвий, надлишковий, надмірний; звільнений, скорочений.

Redundant constraint зайва [надлишкова] в’язь.

Redundant system (*син.* hyperstatic system, statically indeterminate system) статично невизначувана система.

Refem line лінія відліку.

Reference посилання, згадування; співвідношення; еталон; довідковий; подавати примітки; знаходити за посиланням, довідуватися.

Reference frame (*син.* reference system) система відліку.

Reference pitch line of a rack ділильна пряма рейки.

Relation відношення; залежність; зв'язок.

Relationship співвідношення, залежність.

Relative відносний; порівняльний; взаємний; пов'язаний один з одним; відповідний.

Relative displacement відносне зміщення.

Relative motion відносний рух.

Relaxation релаксація, ослаблення, розслаблення; пом'якшення; зменшення напруження.

Reliability (*син.* durability) надійність.

Render віддавати належне; відтворювати, зображати; виконувати (*роль*); перекладати (*на іншу мову*); приводити до певного стану; топити (*сало*).

Rendered fat топлене сало.

Repeated повторний; частий.

Repeated loadingциклічне навантаження.

Repetition багаторазовість; повторення.

Repetitive повторюваний.

Represent зображати; втілювати, уособлювати, являти собою.

Reproduction відтворення.

Resistance опір, протидія.

Resistance force сила опору.

Resistance to rolling тертя кочення.

Respond відповідати, відгукуватися, реагувати (*to*).

Responsibility відповідальність.

Responsible відповідальний, важливий.

Resonance резонанс.

Rest (upon) опиратись (на).

Restyling (*син.* part restyling) зміна конструкції деталі.

Resultant рівнодійна, головний вектор.

Resultant vector головний вектор.

Return повернення.

Return angle (*син.* fall angle) кут повернення.

Reversal реверсування, зміна напрямку на протилежний.

Reversibility властивість оберненості.

Reversing movement зміна напрямку руху.

Right-angled triangle прямокутний трикутник.

Right-angle bevel gearing ортогональна конічна передача.
Rightmost position крайнє праве положення.
Rigid body абсолютно тверде тіло.
Rigidity жорсткість.
Rigidly bound жорстко зв’язаний.
Ring кільце, обід.
Ring gear зубчастий вінець.
Ring pivot кільцева п’ята.
Rise підніматися, вставати; підноситися (над чимсь - *above*); підвищуватися, збільшуватися; підвищення, піднесення; підйом, підняття; збільшення; височина, пагорб.
Rise angle кут віддалення.
Rise phase фаза віддалення.
Rivet(ed) joint (*син***. riveting)** заклепкове з’єднання.
Rocker коромисло, балансир, шатун, хитний важіль, куліса.
Rocker-and-crank mechanism кривошипно-коромисловий механізм.
Rocker follower (*син.* **rocker, swinging follower)** хитний (поворотний) штовхач.
Rod стержень, брус; шток, тяга.
Roll обертання; хитання; вал, барабан, циліндр; вальці; крутити(ся); обертати(ся); перекочувати(ся); повертати(ся); прокатувати; вальцювати, плющити (метал).
Roll over перекочуватись, перевертатись.
Roll over each other перекочуватись один по одному.
Roller ролик.
Roller follower штовхач, споряджений роликом.
Rolling катання, прокатування, вальцювання; обертальний; хитний; роликовий, котковий.
Rolling damper котковий гасник.
Rolling friction тертя кочення.
Rolling friction with slippage тертя кочення з проковзуванням.
Rolling mill прокатний стан.
Root корінь, причина, джерело; корінь (*мат.*); ніжка зуба зубчастого колеса
Root circle (*син.* **dedendum)** коло западин.
Root of a tooth (*син.* **root, dedendum)** ніжка зуба зубчастого колеса.
Root radius радіус кола западин.
Rotating обертальний, поворотний.
Rotating sense напрямок обертання.
Rotation обертання; періодичне повторення; чергування.
Rotation angle кут повороту.
Rotational axis вісь обертання.
Rotor ротор.
Roughness (*син.* **asperity)** нерівність; шорсткість.
Equilibrium roughness рівноважна шорсткість.
Roughness parameter параметр шорсткості.
Rounded закруглений.
Route курс, напрямок; засіб, шлях (*перен.*).
Routine заведений порядок; узвичаєна практика; певний режим; шаблон.
Routine problem типова задача.
Rubbing тертя; стирання.
Rubbing bodies тіла, що труться.
Rubbing path (*син.* **sliding distance)** шлях тертя.
Rubbing surface поверхня тертя.
Ruled surface лінійчата поверхня.
Run біг; пробіг (локомотива, вагона), відрізок шляху, прогін (залізниці); політ, переліт, рейс, відстань, яку пролітає літак; хід, робота, дія (машини, двигуна); триваючий, неперервний; бігти, крутитись, обертатись; працювати, функціонувати; рухати, переміщати.
Run down зупинятися (про машину).
Run-up пуск.
Running хід, робочий хід, робочий стан (машини); неперервний, послідовний.
Running-in process процес приробки.
Running-in time час приробляння.
Run-in surface прироблена поверхня.

S

Saltatory variation стрибкоподібна зміна.
Scale масштаб.
Scale factor масштабний коефіцієнт.
Scale mark мітка шкали.
Scaling magnitude масштабне значення.
Scratching дряпання, насічка.
Screw гвинт, шнек, черв'як; гвинтовий; загвинчувати; нарізати різьбу.
Helical screw гвинт.
Screw axoids of relative motion гвинтові аксоїди відносного руху.
Screw driving загвинчування.
Screw gear (*син.* **spiral gear)** гвинтове зубчасте колесо, гвинтова зубчаста передача.
Screw motion (*син***. helical motion)** гвинтовий рух.
Screw pair гвинтова пара.
Screw-nut gear передача гвинт-гайка.
Screw-thread різьба, гвинтова нарізка.
Secant plane січна площина.
Section переріз; відрізок; сегмент, частина; параграф; розділ книги; ділити на частини.
Sector сектор, частина; дільниця, куліса; поділяти на сектори.
Secure забезпечувати, гарантувати безпеку; одержувати, діставати, здобувати; безпечний, в безпеці; гарантований.
Seizure заїдання, заклинювання; захват, захоплення.
Seizure wear знос при заїданні.
Self-braking condition умова самогальмування.
Semisolid lubricant пластичний мастильний матеріал.
Separate окремий, ізольований; відокремлений; особливий; індивідуальний, самостійний; відокремлювати(ся), відділяти(ся), розділяти(ся); розкладати (на частини).
Separate out виділяти, від’єднувати.
Separation відділення.
Sequence послідовність; порядок, ряд; наслідок, результат.

Sequential послідовний.
Serial positions послідовні положення.
Series серія, ряд, група, послідовне з'єднання.
Series coupling (*син.* series connection) послідовне з'єднання.
Series-multiple coupling (connection) змішане з'єднання.
Set-point заданий.
Several кілька; кожний, окремий; свій, власний, індивідуальний.
Severe несприятливий.
Shaft вал.
Shaft angle між вісний кут (*конічна передача*).
Shaded заштрихований.
Shaping cutter довбач.
Sharp різкий, гострий, гострокінцевий, вигострений; різко, раптово, круто.
Shift зміщення, зсув, перемикання (*швидкостей*); переміщати(ся); пересувати(ся).
Shift factor (*син.* correction factor for profile shift) коефіцієнт зміщення.
Shock absorber амортизатор, буфер.
Side бік, сторона, край; гілка (*паса*).
Silent action безшумна дія.
Similar подібний, адекватний.
Similarity подібність.
Simultaneously одночасно, разом, спільно.
Single gearing (*син.* two-gear train) одноступінчаста зубчаста передача.
Single-enveloping worm (*син.* cylindrical worm) циліндричний черв'як.
Single-threaded однозахідний (черв'як, гвинт).
Sinusoid синусоїда.
Skew косий; асиметричний; схил, нахил; відхилятися, перехрещуватись (про мимобіжні осі); перекошувати(ся).
Skill майстерність, уміння; вправність.
Slackening притуплення, ослаблення, виснаження, спад.
Slide ковзання; рівний, розмірений хід (*машини*); спускний жолоб, похила площина; зсув; ковзна частина (машини); ковзати, засовувати, всовувати.
Slider (*син.* crosshead, cylinder piston, sliding block) повзун.
Slider-crank mechanism кривошипно-повзунковий механізм.
Sliding ковзання, проковзування.
Sliding block (*син.* crosshead, cylinder piston, slider) повзун.
Sliding cam (*син.* wedge cam) пересувний (клинчастий) кулачок.
Sliding distance (*син.* rubbing path) шлях тертя.
Sliding follower (*син.* pusher, translating follower) поступально рухомий штовхач.
Sliding friction (*син.* kinetic friction) тертя ковзання.
Sliding motion рух із ковзанням, ковзання.
Sliding pair поступальна пара.
Sliding thrust bearing підп'ятник ковзання.
Slideway спрямовуюча (верстата).
Slight displacement мале переміщення.
Slip velocity швидкість ковзання.
Slippage проковзування.
Slope ratio (*син.* slope) тангенс кута нахилу.
Smooth curve гладка крива.
Smoothed (*син.* stepless) плавний.
Smoothness плавність.
Operation smoothness плавність роботи.
Socket гніздо, заглиблення, западина, муфта; патрубок.
Solid твердий; суцільний; масивний; тривимірний, просторовий, кубічний; тверде тіло (*фіз.*).
Solid greasing substance тверда змащувальна речовина.
Solid lubricant тверде мастило.
Solid oil солідол.
Somewhat почасти, до деякої міри; щось, дещо.
Source джерело, першопричина, початок.
Source activity активність джерела.
Spall уламок, тріска, скалка; відколювати.
Space motion просторовий рух.
Spalling викрашування (контактних поверхонь), розтріскування, відшарування.
Gear teeth spalling викрашування зубчастої передачі.
Spanning angle кут обхвату (охоплення).
Spatial просторовий.
Spatial force system просторова система сил.
Spatial gearing просторове зубчасте зачеплення, просторова зубчаста передача.
Specific спеціальний, особливий, конкретний; певний, точний; характерний; питомий.
Specific pressure (*син.* unit pressure) питомий тиск.
Specific pressure ratio коефіцієнт питомого тиску.
Specific sliding ratio коефіцієнт питомого ковзання.
Specific weight густина (матеріалу).
Specifically зокрема.
Spectrum спектр, діапазон.
Speed of rotation частота обертання.
Spend витрачати, тратити, затрачувати.
Sphere сфера, куля.
Spheric(al) сферичний.
Spherical mushroom follower сферичний грибоподібний штовхач.
Spiral gear (*син.* screw gear) гвинтове зубчасте колесо, гвинтова зубчаста передача.
Spline connection (*син.* slip joint) шліцьове з'єднання.
Spontaneous спонтанний, мимовільний, безпосередній, невимушений; стихійний.
Spontaneously спонтанно, самочинно, мимовільно.
Spring пружина, ресора; пружність, еластичність; відскік, випрямлення.
Spring constant жорсткість пружини.
Spur виступ, шип, зуб, наконечник.
Spur gear прямозубе циліндричне колесо, прямозуба циліндрична передача.
Spur gearing (*син.* spur gear) прямозуба циліндрична передача.
Square thread прямокутна різьба.
Stable equilibrium стійка рівновага.

Standard стандартний.
Standard pitch circle (*син.* nominal pitch circle) ділильне коло.
Standard pitch radius (*син.* nominal pitch radius) радіус ділильного кола, ділильний радіус.
Standardized стандартизований.
Standing що стоїть, нерухомий; не пересувний; стояння.
Standing axis вертикальна вісь.
Staple головний елемент (чогось), головний, основний
Start
1. починати; братися (за щось); пускати (машину); пуск в хід; рушання з місця.
2. **(*син.* run)** розбіг, розгін (машини, агрегату).

Static(al) статичний, стаціонарний, нерухомий**.**
Static friction (*син.* stiction) тертя спокою.
Statics статика.
Stationary нерухомий.
Stationary state нерухомий стан.
Steady стійкий; міцний, твердий; рівний; сталий; рівномірний; постійний, незмінний; ставати твердим (стійким); опора, люнет.
Steady speed постійна швидкість.
Steady regime (*син.* steady run, steady state mode) усталений режим.
Stepless (*син.* smoothed) плавний, безступінчастий.
Stick-slip переривчасте переміщення; стрибкоподібна подача.
Stiffly precision instrument високо точний прилад.
Stipulated фіксований, обумовлений.
Stop
1. зупиняти(ся); припиняти(ся); закінчувати(ся); зупинка, затримка, припинення; кінець; пауза, перерва; обмежник, стопор.
2. **(*син.* running-out)** вибіг (машини, агрегату).

Store накопичувати.
Strain-hardening зміцнювальний.
Stream потік, струмінь, напрямок, рух.
Strength міцність.
Strengthening treatment зміцнювальна обробка
Stretch розтягувати(ся), витягувати(ся), подовжувати,тягти(ся, натягувати(ся); витягання, розтягання, подовження.
Strict точний, певний.
Strictly точно, визначено, без відхилень.
Structural структурний; будівельний.
Structural damping конструкційне демпфірування.
Structural group (*син.* Assur group, structure group) структурна група, група Ассура.
Structure структура; будова; будівля, споруда.
Structure chart структурна схема.
Structure synthesis структурний синтез.
Successive values послідовні значення.
Suitable підхожий; що відповідає; придатний.
Sum сума, підсумок; складати.
Sum total загальний підсумок (сума).
Summand доданок.
Sun gear сонячне колесо.
Superficial уявний; поверховий, неглибокий, зовнішній.
Superficial coefficient of friction зведений коефіцієнт тертя.
Superpose суміщати, накладати.
Superposed суміщений.
Support reaction опорна реакція.
Support point точка опори.
Suppress класти край, стримувати, придушувати.
Suppression стримування, придушення, заборона.
Surface поверхня; обробляти поверхню, покривати поверхню (with - чим-небудь.).
Surface abrasion абразивний знос поверхні.
Surface bed поверхневий шар.
Surface capacity поверхнева міцність.
Surface condition стан поверхні, чистота поверхні, якість поверхні.
Surface deterioration знос поверхні.
Surface finish фінішна обробка поверхні; шорсткість; квалітет поверхні, якість обробки поверхні.
Surface roughness (*син.* as-turned finish) шорсткість поверхні.
Surface treatment (*син.* surfacing) обробка поверхні.
Surfacing (*син.* surface treatment) обробка поверхні.
Surfent of area елементарна площадка.
Sweeping (sweep) angle of involute кут розгортання евольвенти.
Swing коливати(ся), гойдати(ся); хитати(ся); гойдання, хитання; коливання; амплітуда коливань.
Swinging гойдання, хитання, коливання; поворотний.
Swinging arm хитний важіль, куліса.
Swinging follower (*син.* oscillating follower, rocker, rocker follower) хитний (поворотний) штовхач.
Swing link хитний важіль.
Swinging motion (*син.* swinging movement) зворотно-обертальний (хитальний, поворотний) рух.
Synthetic синтетичний, штучний.
Synthetic oil синтетичне мастило.

T

Tangent дотична; тангенс; дотичний
Tangent to circle дотична до кола.
Tangent to curve дотична до кривої.
Tangent point (*син.* point of tangency, contact point) точка дотику.
Taper (*син.* become pointed/sharp; become more acute) загострюватись.
Tangential acceleration тангенціальне прискорення.
Tear вирив, виривання, знос.
Tear(ing) (*син.* attrition, deterioration, wear, wear-out) знос, зношування.
Teflon фторопласт, тефлон.
Tend to infinity прямувати до нескінченності.
That is (*син.* i.e., that is to say, in other words) тобто.
Theoretical line of action (*син.* pressure line) теоретична лінія зачеплення.

Theoretically (*син.* **in theory)** теоретично.
Thicken потовщуватись.
Thickening temperature температура згущення; температура застигання (*мастила*).
Thin-film lubrication (*син.* **borderline lubrication)** граничне змащення.
Thread різьба, нитка.
Angle of thread кут профілю різьби.
Square thread прямокутна різьба.
Threaded connection (*син.* **threaded fastener, screw joint, threaded joint)** різьбове з'єднання.
Thread pitch крок різьби.
Triangular thread трикутна різьба.
Through через, завдяки.
Thrown over перекинутий через.
Tightening (*син.* **torqueing)** затяжка, затягування.
Tighting force сила натягу.
Tilt похиле положення; нахил; нахиляти(ся).
Tilt angle кут нахилу.
Tilted нахилений.
Time час; термін, строк; життя, вік; раз; періодичний; призначати час.
Time axis вісь часу.
Time derivative похідна по часу.
To decompose a force розкласти силу.
To meet in a point перетинатись в точці.
To prove a theorem довести теорему.
Tolerance допуск.
Tool інструмент.
Toolkit комплект [набір] інструментів.
Tooling method спосіб (технологія) механічної обробки.
Tooth зуб, зубець.
Tooth gear зубчаста передача.
Tooth interference інтерференція зубчастих профілів
Tooth profile (*син.* **tooth form)** профіль зубця.
Tooth surface бічна поверхня зубця.
Tooth-wheel зубчасте колесо.
Toothed зубчастий.
Toothed gear зубчасте колесо; зубчаста передача.
Toothed gearing (*син.* **toothed drive)** зубчаста передача.
Toothed rack зубчаста рейка.
Toothed wheel зубчасте колесо, шестерня; храпове колесо; храповик.
Toothing зачеплення.
Gear toothing зубчасте зачеплення.
Top speed гранична (максимальна) швидкість.
Torqueing (*син.* **tightening)** затяжка, затягування.
Touch дотикатись.
Touch on впливати; мати відношення; досягати; зачіпати, торкатися побіжно.
Total повний, цілковитий, абсолютний; тотальний; весь, цілий; сукупний, сумарний; ціле, сума; підбивати підсумки, підраховувати.
Total acceleration повне прискорення.
Total cam angle of rotation повний кут повороту кулачка.
Total work сумарна робота.
Track напрямний пристрій; слід, колія.
Track cam (*син.* **face cam)** пазовий кулачок.
Transfer перенесення; переміщення; переносити, переміщати (з - *from*, в - *to*).
Transfer function of a mechanism передатна функція механізму
Transfer of material перенос матеріалу.
Transient regime неусталений режим.
Transition curve крива спряження, перехідна крива.
Translation зміщення, зсув, переміщення; переклад, тлумачення.
Translation movement (*син.* **translation motion)** прямолінійний рух; поступальний рух.
Translating follower (*син.* **pusher, sliding follower)** поступально рухомий штовхач.
Transmission ratio (*син.* **reduction ratio)** передатне число.
Transportation motion переносний рух.
Transverse поперечний; той, що перетинається.
Transverse circular pitch (*син.* **circular pitch in plane of rotation)** торцевий крок.
Transverse contact ratio коефіцієнт торцевого перекриття.
Transverse module торцевий модуль.
Transverse section поперечний розріз, поперечний переріз.
Trapezoids (*син.* **trapezium)** трапеція.
Traverse speed швидкість переміщення.
Triangle трикутник.
Triangular thread трикутна різьба.
Tribological conditions умови тертя.
Trigonometric transformation тригонометричне перетворення.
Triple arm group (*син.* **triad)** триповідкова група (тріада).
Trundle цівка, цівкова шестерня.
Turn крутити(ся); повертати(ся), обертати(ся); оберт (колеса); поворот; зміна напряму.
Turn of thread виток різьби.
Turning обертання; обточування; токарна робота; токарне ремесло; перетворення; токарний; що обертається; обертовий; поворотний.
Turning moment обертальний момент.
Turning movement обертальний рух.
Turning operation точіння; обточка.
Turning pair (*син.* **hinge)** обертальна пара.
Turning speed частота обертання.

U

Unbalance (*син.* **imbalance)** неврівноваженість.
Unclosed kinematic chain незамкнений кінематичний ланцюг.
Undamped недемпфований.
Undercutting підрізання.
Undesirable небажаний, незручний, непридатний, непідхожий.
Unfavourable несприятливий.
Unfavourable effect несприятливий вплив (дія).
Unfavourable position несприятливе положення.

Uniform одноманітний; однорідний; постійний, сталий.

Uniform acceleration рівномірне прискорення, рівномірно прискорений рух.

Uniform distribution рівномірний розподіл.

Uniform motion рівномірний рух.

Uniform rotation рівномірне обертання.

Uniformly accelerated motion рівноприскорений рух.

Unique однозначний; однозначно визначуваний (*мат.*); унікальний.

Uniquely defined (*син.* **unique)** однозначно визначуваний.

Unit агрегат, секція; вузол; елемент; одиниця; ціле; одиниця вимірювання.

Unit pressure (*син.* **specific pressure)** питомий тиск.

Unlike на відміну від; не схожий на; не такий, як.

Unlocking розмикання, роз'єднання, розчеплення.

Unlubricated friction тертя без змащення, сухе тертя.

Unparted hyperboloid однопорожнинний гіперболоїд.

Unproductive непродуктивний.

Unreliable ненадійний.

Unrun surface неприроблена поверхня.

Usage використання, користування.

Usage conditions умови експлуатації.

Useful корисний, придатний.

Useful load корисне навантаження.

Useful work корисна робота.

Unstable нестабільний, мінливий, нетвердий, нестійкий.

Unstable motion (*син.* **hunting, wobble)** нестійкий рух.

Utilization використання, застосування, утилізація.

V

Validation перевірка достовірності (правильності).

Variable змінний, змінна величина.

Vector вектор; векторний.

Vector equation векторне рівняння.

Vector polygon векторний многокутник.

Vector form векторна форма.

Velocity швидкість.

Velocity analogues аналог швидкості.

Velocity diagram план швидкостей.

Velocity gradient градієнт швидкостей.

Vertex angle кут при вершині.

Vertex of triangle вершина трикутника.

Varying змінний.

Vibration вібрація, коливання.

Vibration absorber віброполиначч; демпфер.

Vibration damping демпфірування коливань.

Vibration isolation віброізоляція.

Vibration protection (*син.* **vibroprotection, vibrodamping)** віброзахист.

Vibration suppressor (*син.* **vibroextinguisher**) віброгасник.

Vibration source джерело вібрацій (коливань).

Vibratory displacement вібропереміщення.

Vibroextinguisher (*син.* **vibration suppressor)** віброгасник.

Vibrodamping (*син.* **vibroprotection)** віброзахист.

Vibroprotection (*син.* **vibrodamping)** віброзахист.

Vice versa навпаки, протилежно.

Viscosity в'язкість, липкість, клейкість; тягучість

Dynamic viscosity динамічна в'язкість.

Kinematic viscosity кінематична в'язкість.

Viscosity coefficient (*син.* **coefficient of viscosity)** коефіцієнт в'язкості.

Viscous в'язкий, липкий, клейкий; тягучий, густий.

Viscous damper (*син.* **liquid damper)** рідинний гасник.

Viscous force сила в'язкого зсуву; сила в'язкого тертя.

Viscous lubrication пластичне мастило.

Viscous shear в'язкий зсув.

Viscous shear stress напруження при в'язкому зсуві.

Visual наочний; видимий.

Visualization наочність; візуалізація.

W

Warm теплий, зігрітий, підігрітий; гріти(ся), нагрівати(ся), зігрівати(ся).

Waste зайва витрата; шкода, збиток, втрата; даремно витрачати; спрацьований, непридатний, бракований, зайвий; марний; непотрібний.

Power waste втрати енергії.

Water-resistance водостійкість.

Wear (*син.* **abrasion, attrition, deterioration, tearing, wear-out)** знос, зношування.

Abrasive wear абразивний знос.

Attrition [attritious] wear знос під час приробляння.

Limiting wear граничний знос.

Mechanical wear механічний знос.

Mechanochemical wear корозійно-механічний знос.

Normal wear (*син.* **service wear)** нормальний (експлуатаційний) знос.

Permissible wear допустимий знос.

Run-in wear знос в період приробки.

Seizure wear знос при заїданні.

Wear factor коефіцієнт зносу.

Wear fragment часточка (фрагмент) зношування.

Wear distribution diagram епюра зносу.

Wear mode вид зносу.

Wear rate швидкість зносу.

Wear resistance (*син.* **abrasion resistance, wearlessness)** опір зношуванню, зносостійкість.

Wear-out знос, зношувати(ся).

Wearlessness (*син.* **abrasion resistance, wear resistance)** зносостійкість.

Wedge клин.

Fluid wedge (*син.* **lubricating oil wedge, physical wedge)** мастильний клин.

Wedge cam (*син.* **sliding cam)** клинчастий (пересувний) кулачок.

Wedge gap клиновий зазор.

Wedge-shaped клиновидний.

Wedge-shaped slider клинчастий повзун.

Wedging phenomenon клиновий ефект.

Wet мокрий, вологий; мочити; змочувати, зволожувати.

Wheel

1. Колесо; коло , кружляння, оберт; котити, везти; , кружляти.
2. **(***син.* **wheel gear)** зубчасте колесо, шестірня.

Wheelwork зубчастий механізм (зубчаста передача).

Widespread (*син.* **prevalent)** дуже поширений.

Width ширина.

While час, відрізок часу; доки, в той час як; незважаючи на те, що; тоді як.

Work робота, праця; оброблювання; робота (*фіз.*); робочий; працювати, трудитися; надавати дії (руху); керувати (*машиною тощо*).

Work of driving force робота рушійних сил.

Work of resistance forces робота сил опору.

Work up розробляти.

Working працюючий; робочий; діючий; робота, дія; експлуатація; розроблення.

Working flank (*син.* **active gear tooth flank)** робочий профіль зубця.

Working medium робоче тіло.

Working pitch circle початкове коло.

Working pitch radius радіус початкового кола, початковий радіус.

Working pressure angle кут зачеплення (*відмінний від стандартного*).

Worm (*син.* **worm screw)** черв'як, шнек, черв'ячний гвинт.

Archimedean worm архімедів черв'як.

Involute worm евольвентний черв'як.

Worm-diameter factor коефіцієнт діаметра черв'яка.

Worm gear pair (*син.* **worm-and-worm pair)** черв'ячна пара.

Worm thread sweeping розгортка витка різьби черв'яка.

Worm wheel (*син.* **worm gear)** черв'ячне колесо.

Worm shaft черв'ячний вал, вал шнека.

Worm toothing черв'ячне зачеплення

Worm-gear (*син.* **worm drive, worm gear set, worm gearing)** черв'ячна передача.

Worse гірший, гірше.

ABSTRACT IN UKRAINIAN

«Теорія механізмів і машин: синтез механізмів, тертя, віброзахист» є другою частиною підручника з дисципліни «Теорія механізмів і машин» для студентів спеціальності 131 «Прикладна механіка», спеціалізації «Динаміка і міцність машин» та "Інформаційні системи та технології в авіабудуванні". В цій частині підручника висвітлені питання синтезу різних типів механізмів, розглядаються основи теорії тертя, зносостійкості та віброзахисту механізмів і машин.

Частина 2 підручника містить 5 розділів. Нумерація їх є продовженням нумерації розділів, що увійшли до частини 1 підручника – з восьмого до дванадцятого.

У розділ 8 розглянуті начала синтезу механізмів. Теоретичний матеріал супроводжується прикладами реалізації методів синтезу шарнірно-важільних механізмів, утворення спряжених поверхонь ланок у механізмах з вищими парами.

Розділ 9 присвячений синтезу зубчастих зачеплень, приводиться сучасна їх класифікація. Найбільша увага сконцентрована на евольвентному зачепленні як основному в сучасних механічних зубчастих передачах, його особливостям, методам виготовлення, контролю якості. Наводиться також загальна інформація щодо просторових зубчастих передач, зокрема конічних та гіперболоїдних.

У розділі 10 викладена теорія кулачкових механізмів. Тут приведена класифікація таких механізмів за конструктивною та кінематичною ознаками. Розглядаються аналітичні та графоаналітичні методи їх синтезу та аналізу.

Розділ 11 присвячений висвітленню основ теорії сухого тертя та теорії змащення в кінематичних парах, впливу тертя на роботу машин і механізмів. Крім того, розглядаються питання зносу в кінематичних парах та зносостійкості їх елементів.

У розділі 12 розглянуті питання вібрацій в механізмах, аналізуються основні джерела їх виникнення. Розглянуті також методи віброзахисту механізмів і машин, наведені приклади проектування демпферів, їх конструкція та сфери застосування.

Теоретичний матеріал супроводжується прикладами розв'язання практичних задач для різних типів механізмів. Кожний розділ підручника супроводжується питаннями для самоперевірки студентами знань в процесі вивчення дисципліни.

Підручник містить ряд додатків з довідковою інформацією для використання при розв'язанні практичних задач та при самоперевірці знань та список рекомендованої літератури.